Heat Equation:
Advection vs Diffusion

Filter-bead game was:
1-D flow
2-D diffusion

$$\rho_{cp} \frac{dT}{dt} = K \frac{\partial^2 T}{\partial x_i^2}$$

LAGRANGIAN
Change in T with time in box

Divergence of heat flow between edges of box

Eulerian
Change in T with time

Advection of temp. grad.

Diffusion…(divergence of heat flow). Does this capture demo?

$$\rho_{cp} \left(\frac{\partial T}{\partial x_1} \right) = K \frac{\partial^2 T}{\partial x_1^2} + K \frac{\partial^2 T}{\partial x_2^2}$$

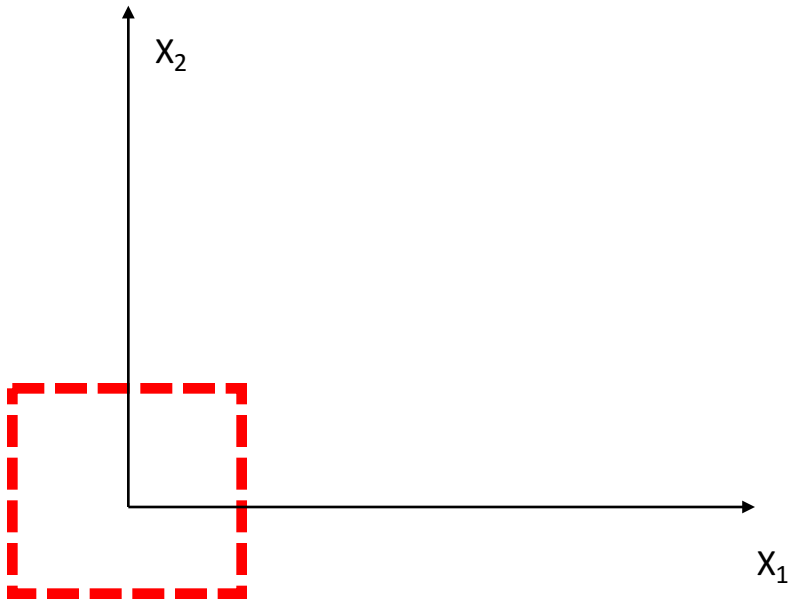
Write on board full equation

1. Did exercise to build intuition for advection-diffusion

1. Wrote math description of advection-diffusion

3. **Now convince me you can switch effortlessly between math terms vs. advection-diffusion patterns**

Here is a small parcel like box, sitting in a much larger domain (let's say it's our stream).



1. Divide into equal teams.

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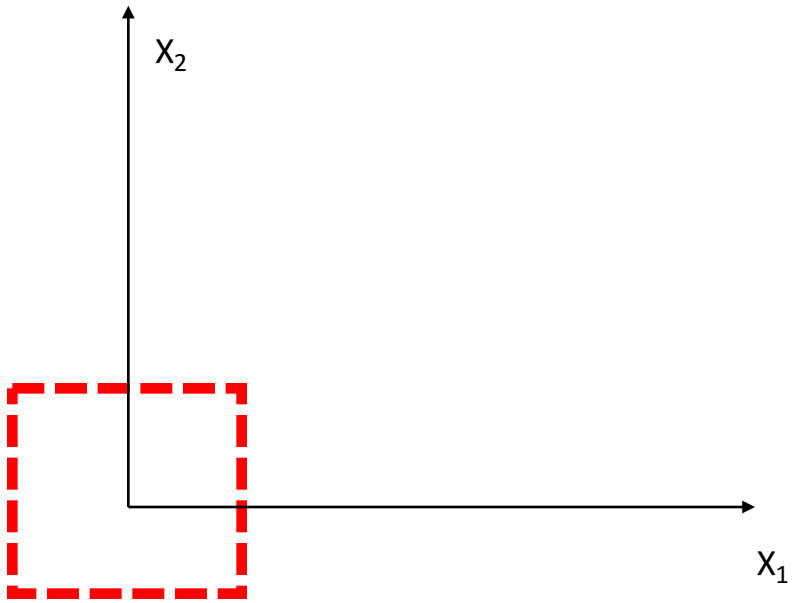

1. Divide into equal teams.
2. I flash up a color contour plot of the local T(x1, x2) field (red-hot, blue-cold)
3. One minute, discuss as a team the sizes of three terms in our PDE

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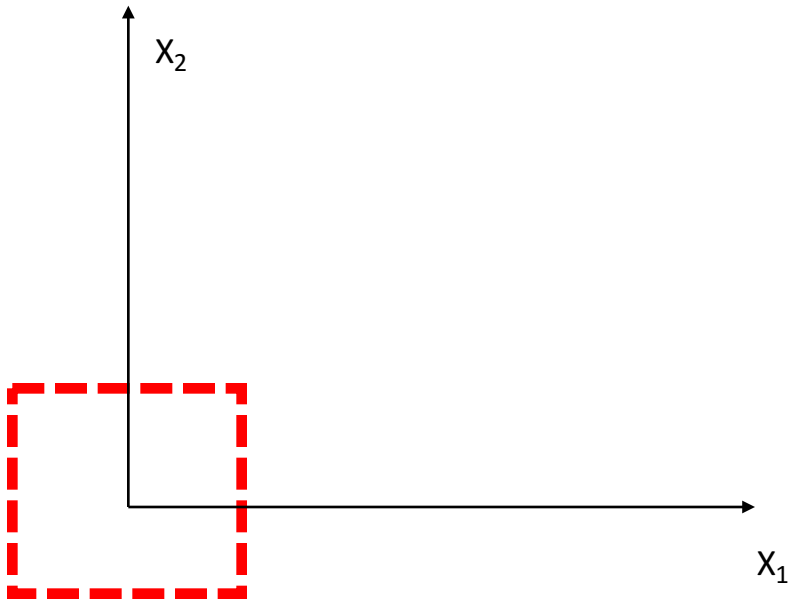


1. Divide into equal teams.
2. I flash up a color contour plot of the local T(x1, x2) field (red-hot, blue-cold)
3. One minute, discuss as a team the sizes of three terms in our PDE
4. One person per team represents magnitude each term
5. Display size by height of your head above floor (zero=sit; big=stand)

Math term stars

$\frac{\partial T}{\partial t}$ Eulerian thermometer: change T in time in spot

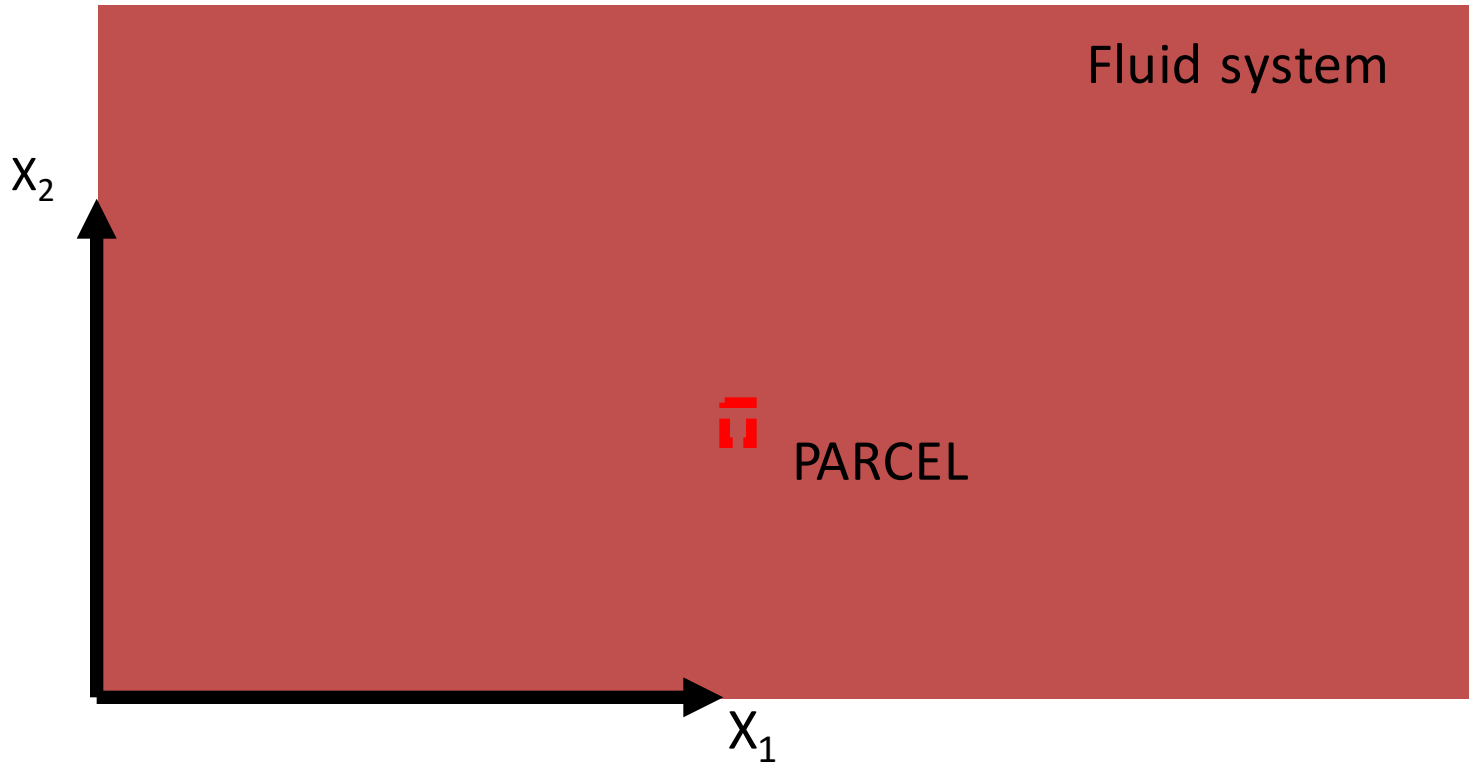
$u_1 \, \partial T/\partial x_1$ Advection in x_1

$u_2 \, \partial T/\partial x_2$ Advection in x_2

$\frac{\partial}{\partial x_1} (K \, \partial T/\partial x_1)$ $K \, \partial^2 T/\partial x^2_1$ Diffusion in x_1

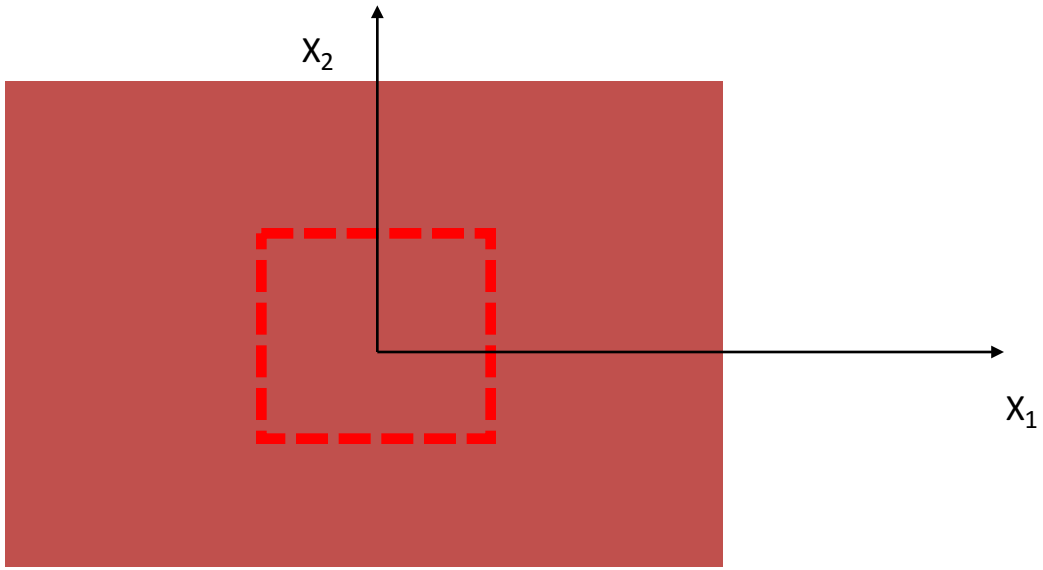
$\frac{\partial}{\partial x_2} (K \, \partial T/\partial x_2)$ $K \, \partial^2 T/\partial x^2_2$ Diffusion in x_2

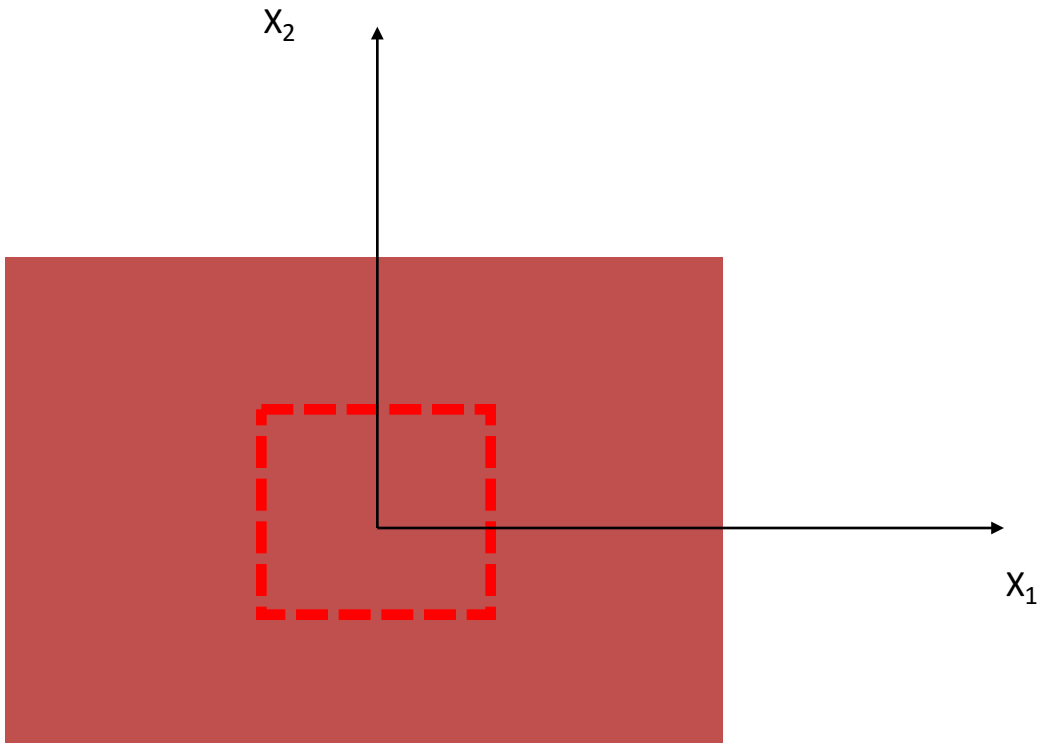
Define a fluid system (like an estuary) as a continuum volume.
Define coordinate axes.
Pick a single parcel (incredibly small section of this volume)



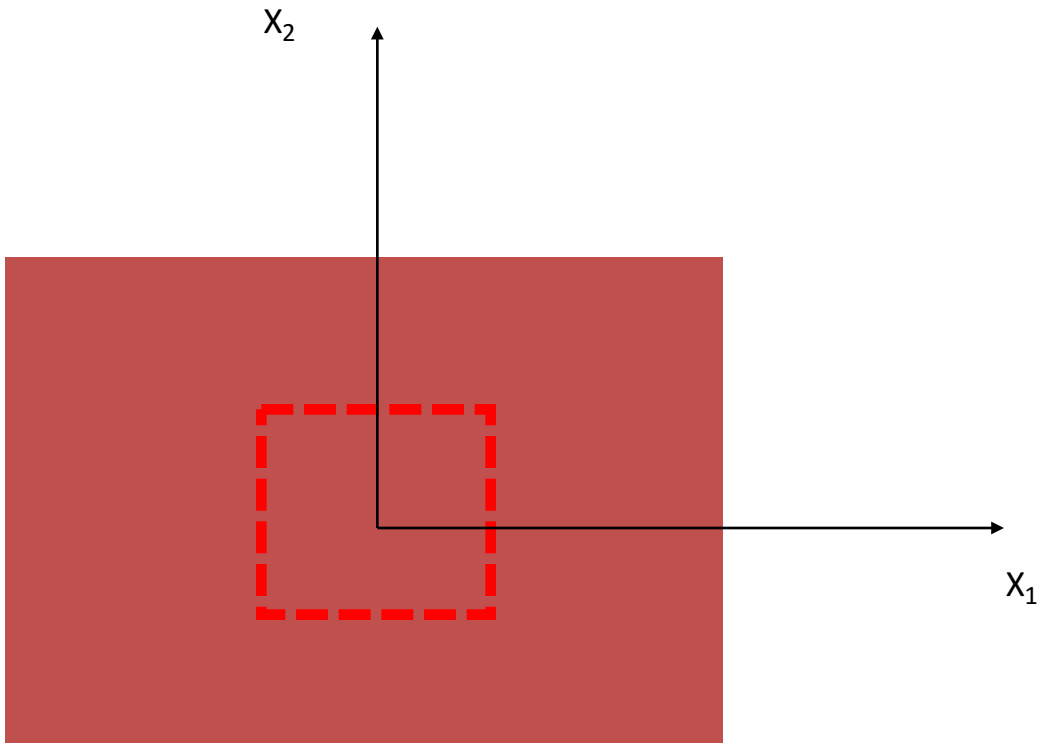
ZOOM IN ON OUR PARCEL.

What's happening in the red bounded box GAME…..





As a group tell me for each term: If your term is **_zero**___, then __**sit**____
a) zero magnitude (sit), b) medium size (squat), c) big magnitude, (stand)



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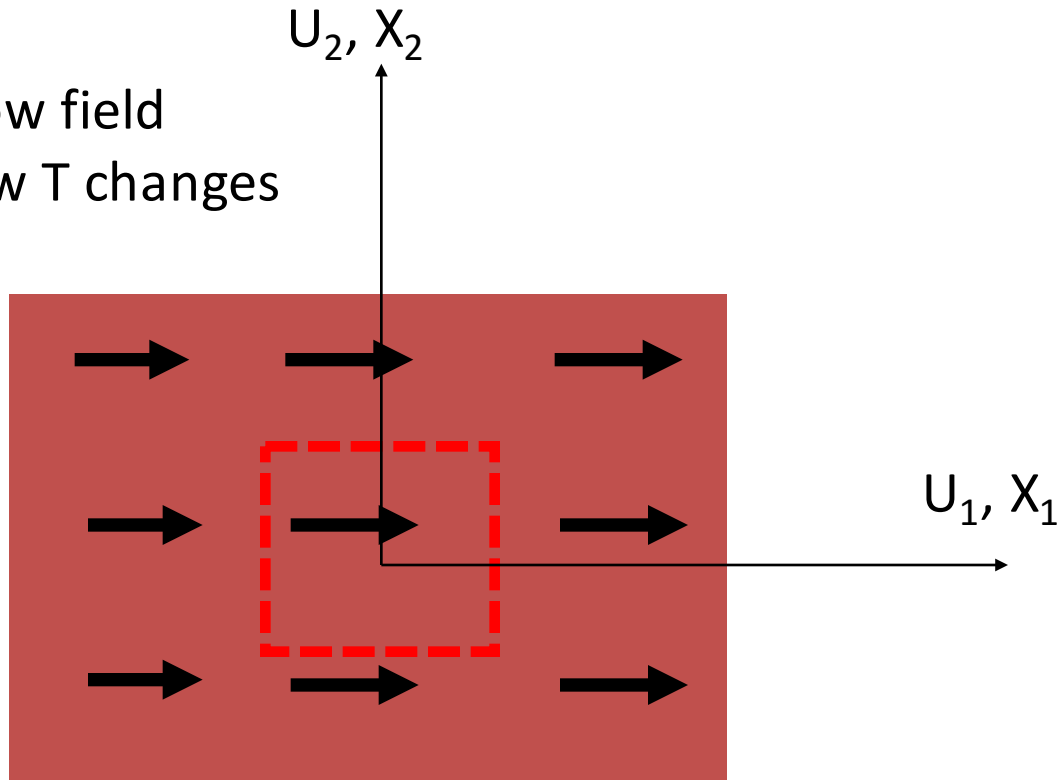
TERMS ARE

$\partial T/\partial t$ $u_1 \partial T/\partial x_1$ $u_2 \partial T/\partial x_2$ $\frac{\partial}{\partial x_1}(K \partial T/\partial x_1)$ $\frac{\partial}{\partial x_2}(K \partial T/\partial x_2)$

Given this flow field
& colors show T changes



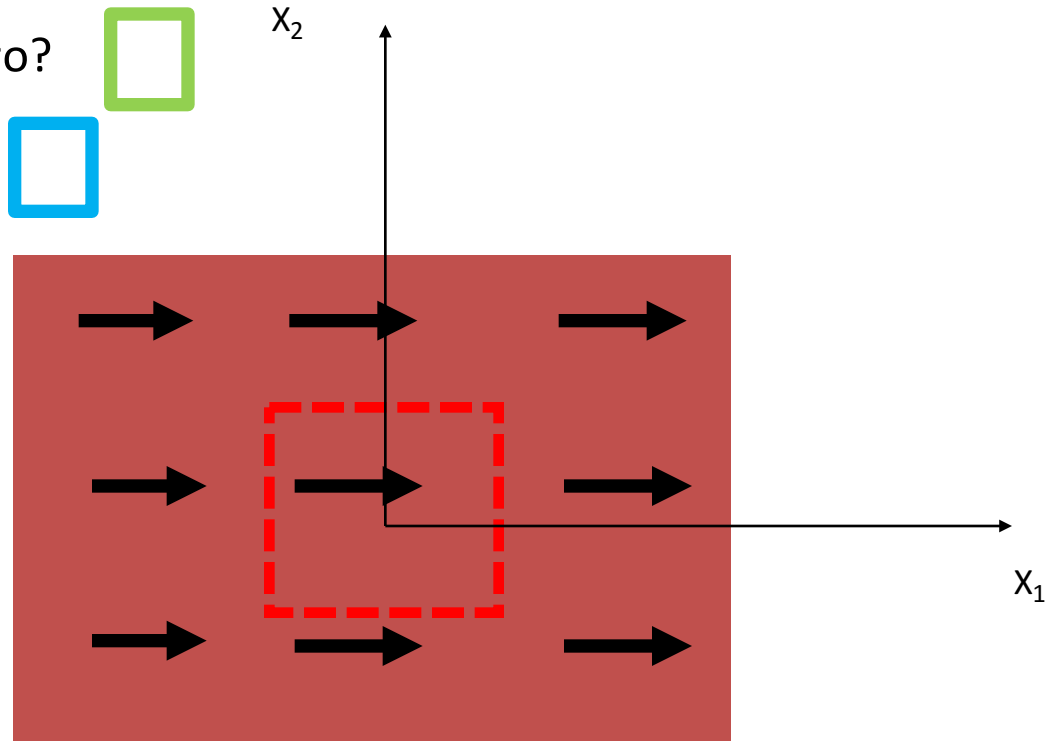
Which terms in math equation zero vs. non-zero?

$\partial T/\partial t$ $u_1\, \partial T/\partial x_1$ $u_2\, \partial T/\partial x_2$ $\frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$ $\frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

ANSWER

Is term in box zero?

Or non-zero?



?

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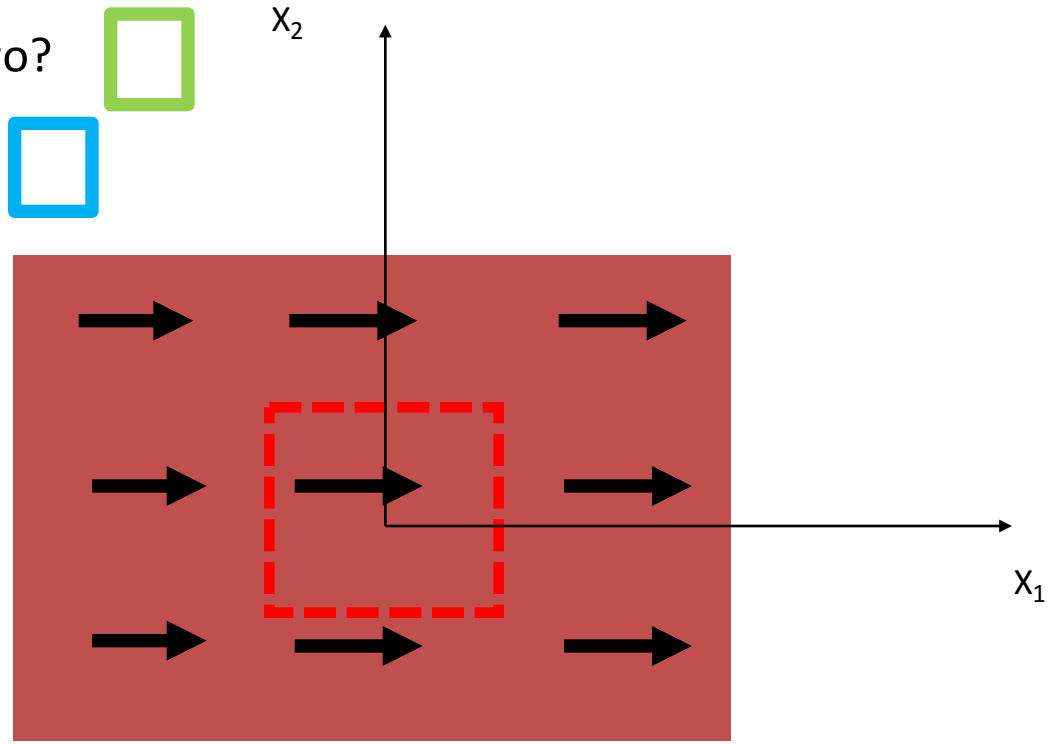

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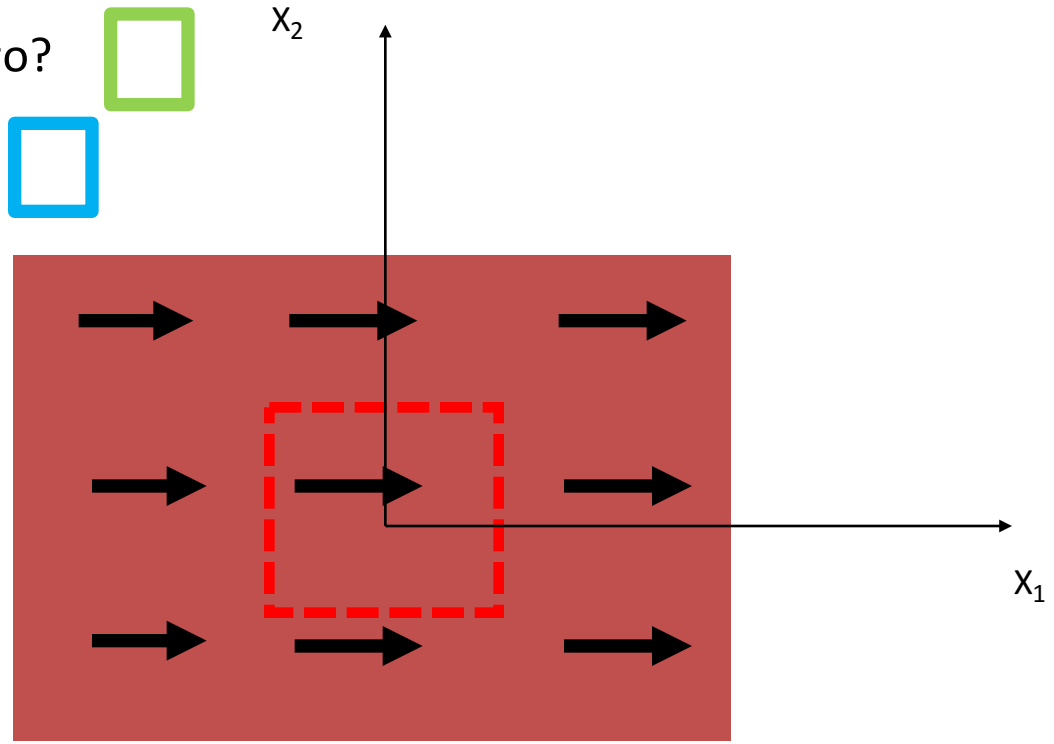

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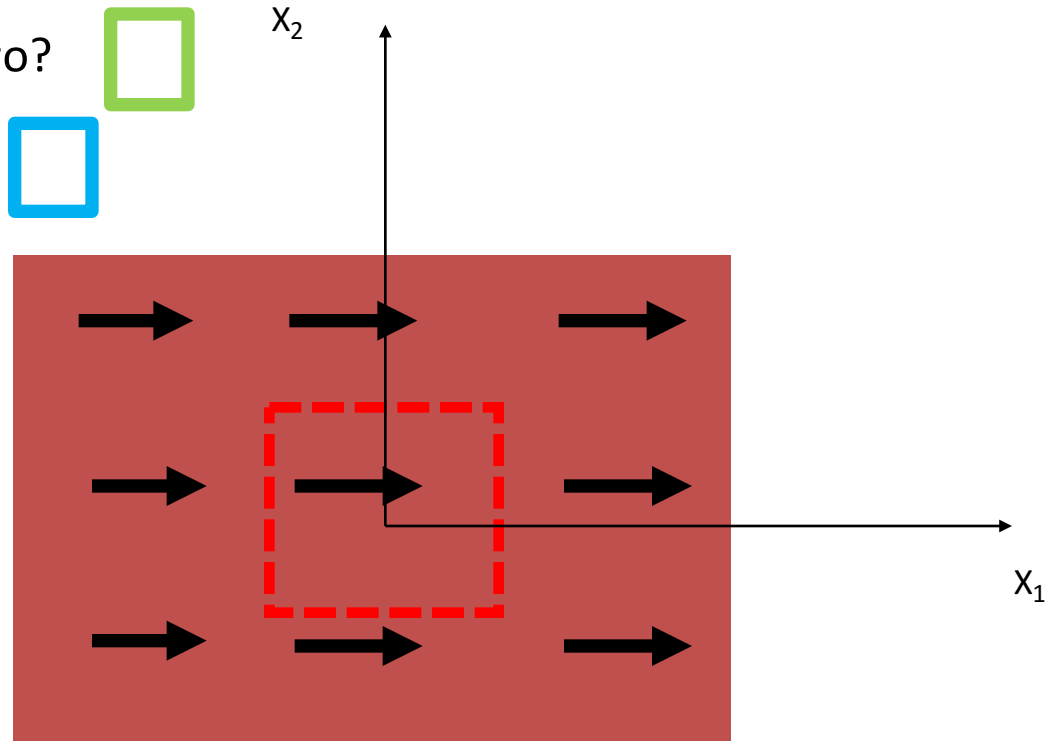

?

$\partial T/\partial t \quad u_1 \; \partial T/\partial x_1 \quad u_2 \; \partial T/\partial x_2 \quad \frac{\partial}{\partial x_1}(K \, \partial T/\partial x_1) \quad \frac{\partial}{\partial x_2}(K \, \partial T/\partial x_2)$

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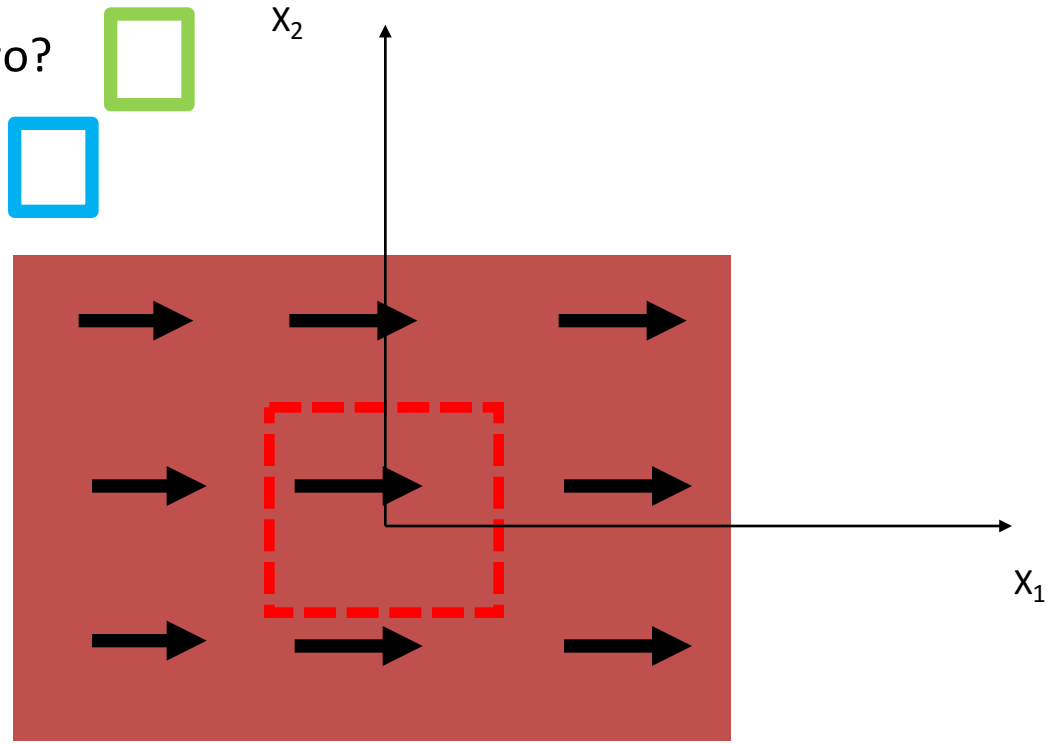

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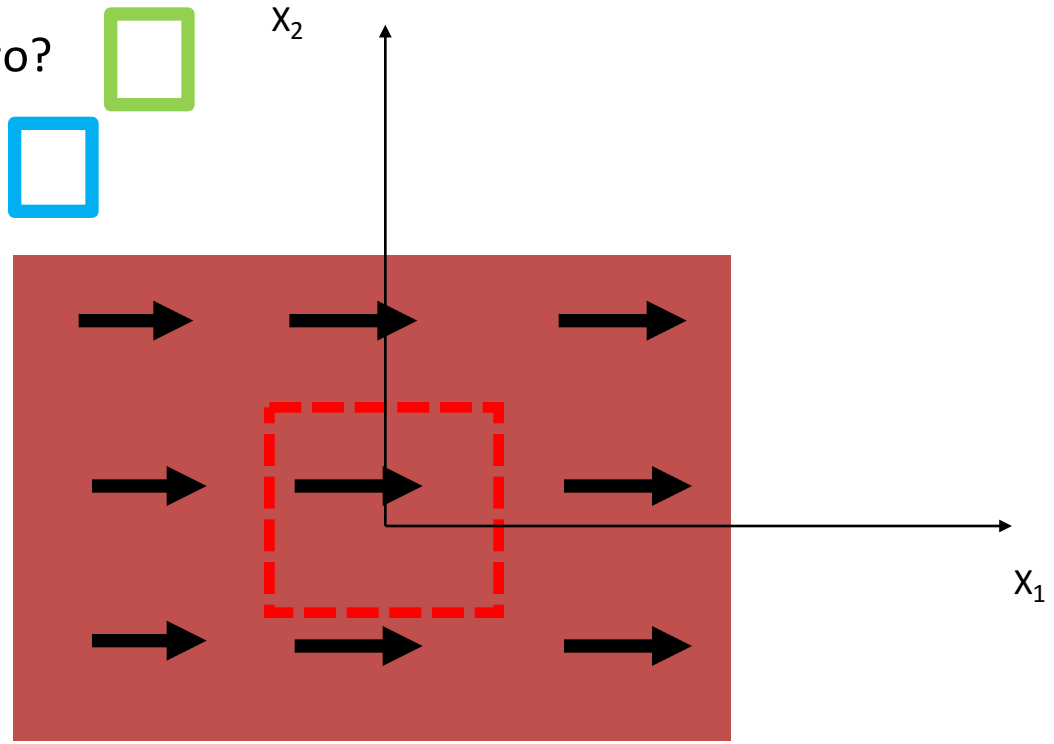

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ANSWER

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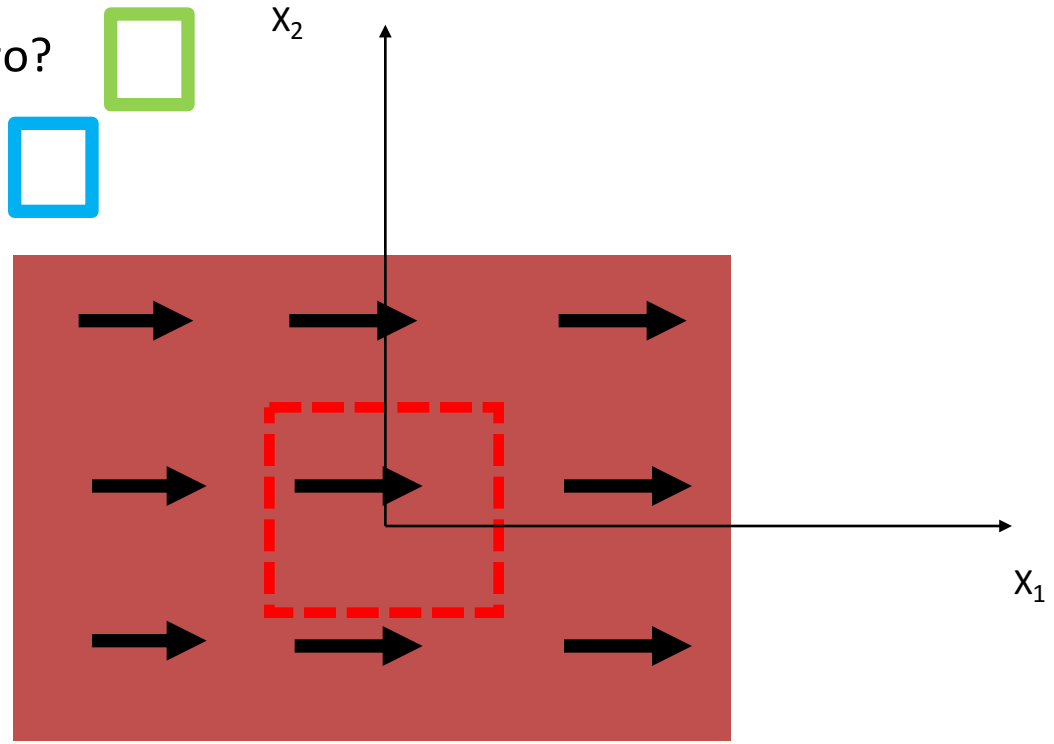

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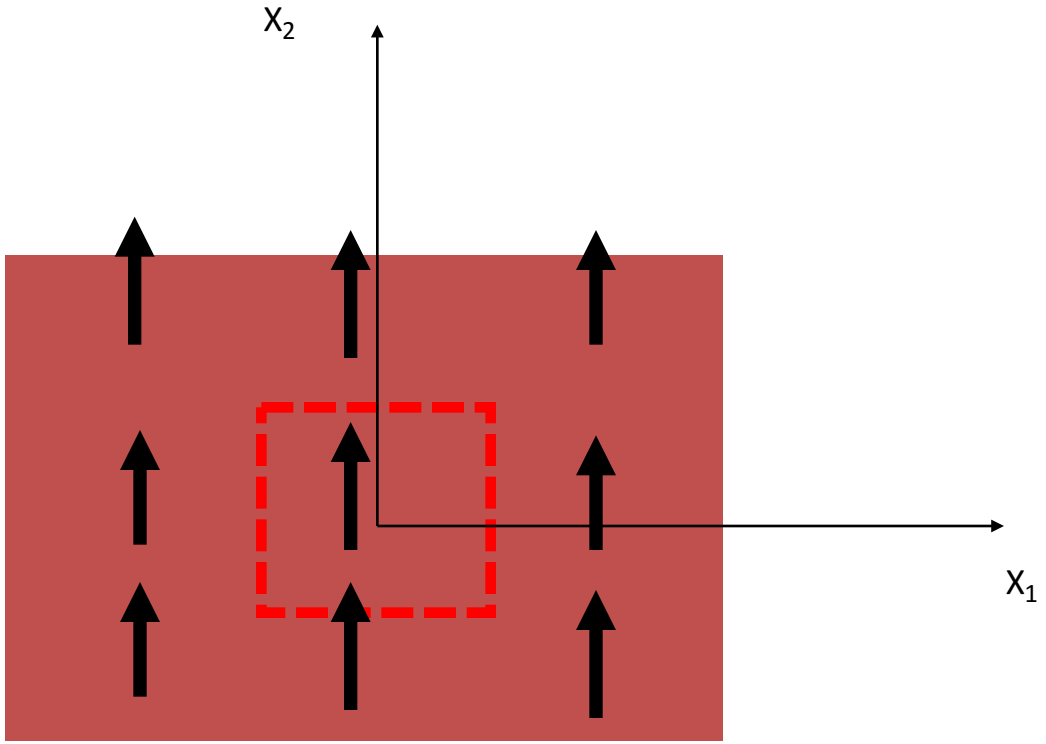
Which terms zero, non-zero?



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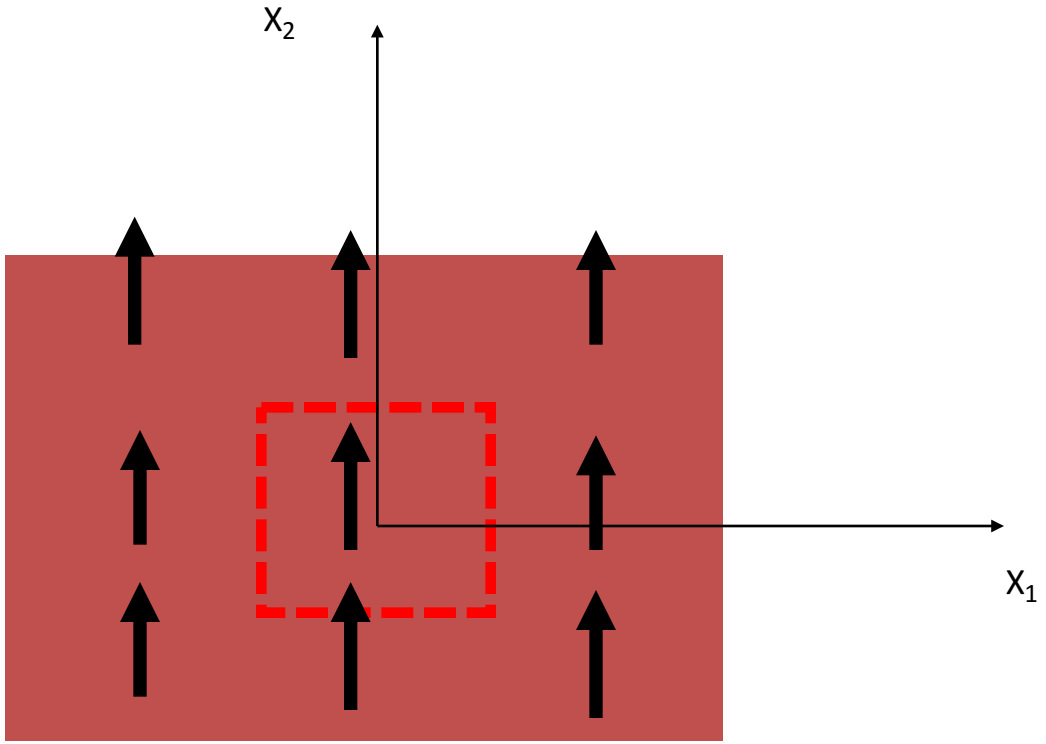

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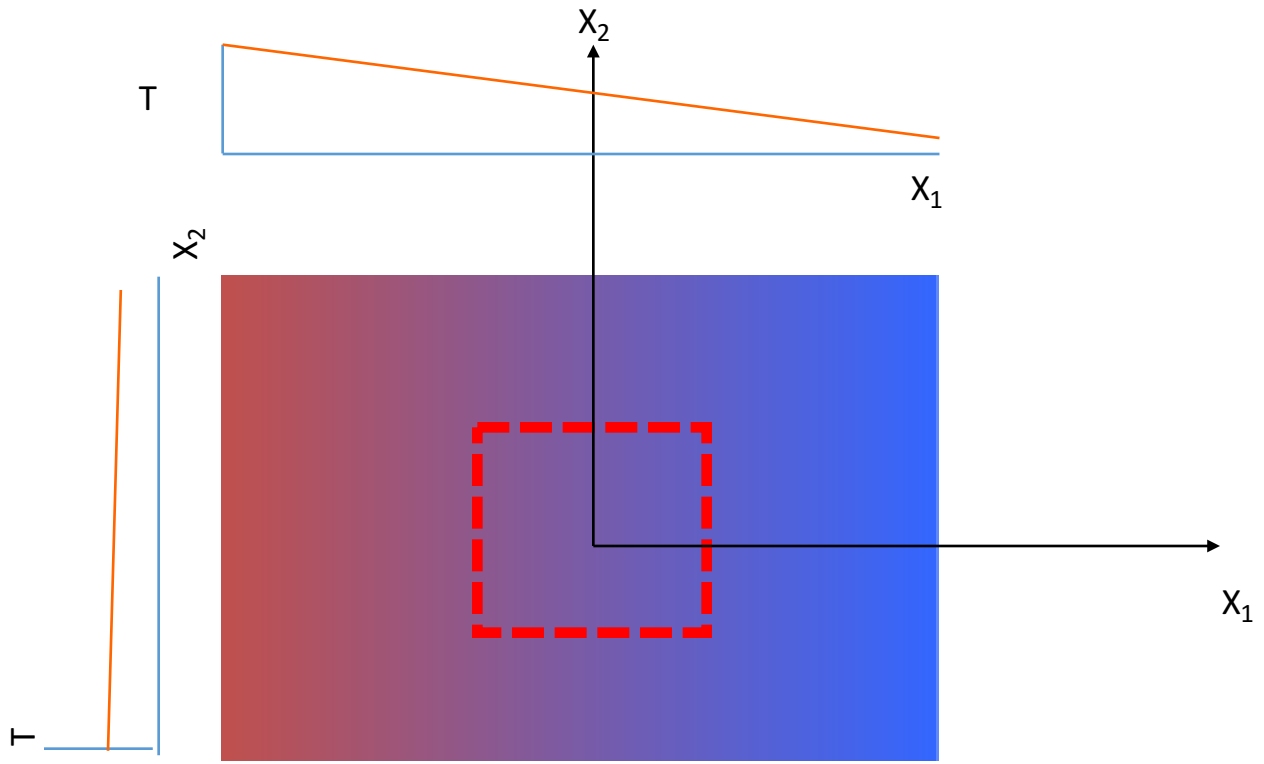
What's happening now in the red bounded box?



?

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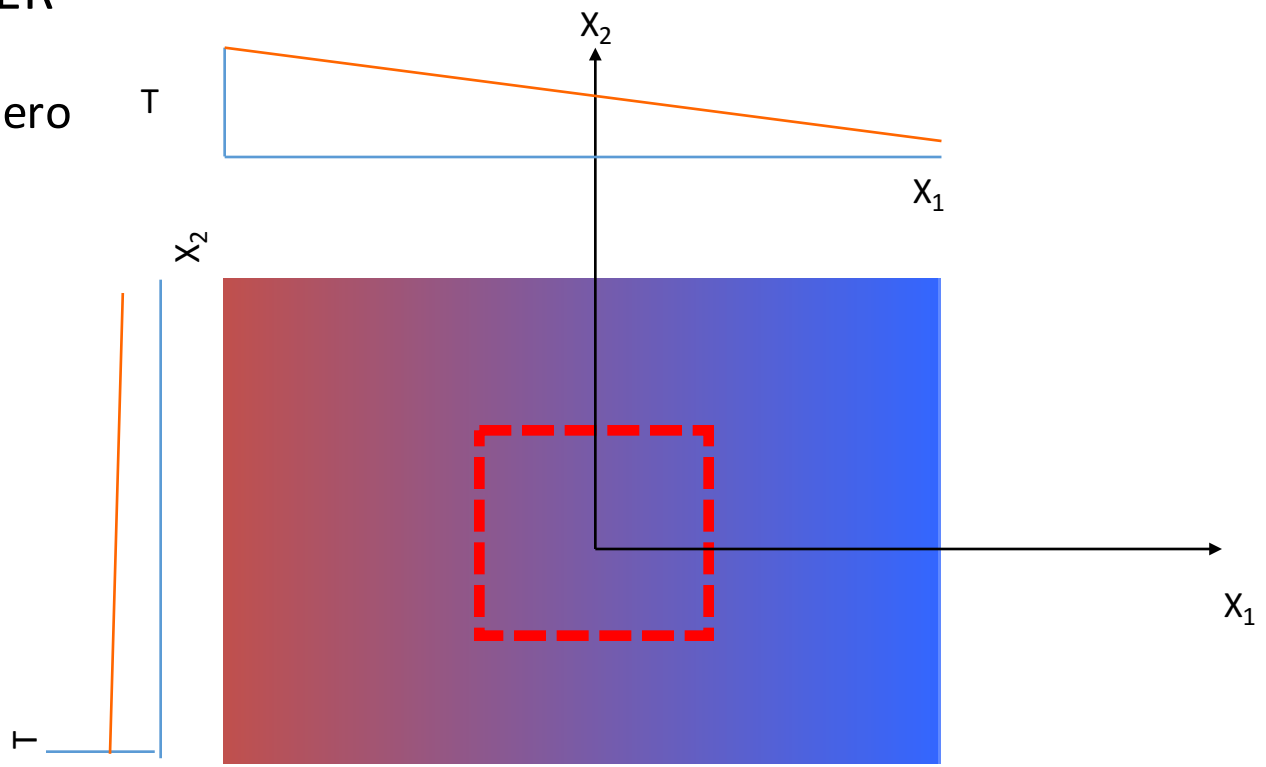
ANSWER

= zero



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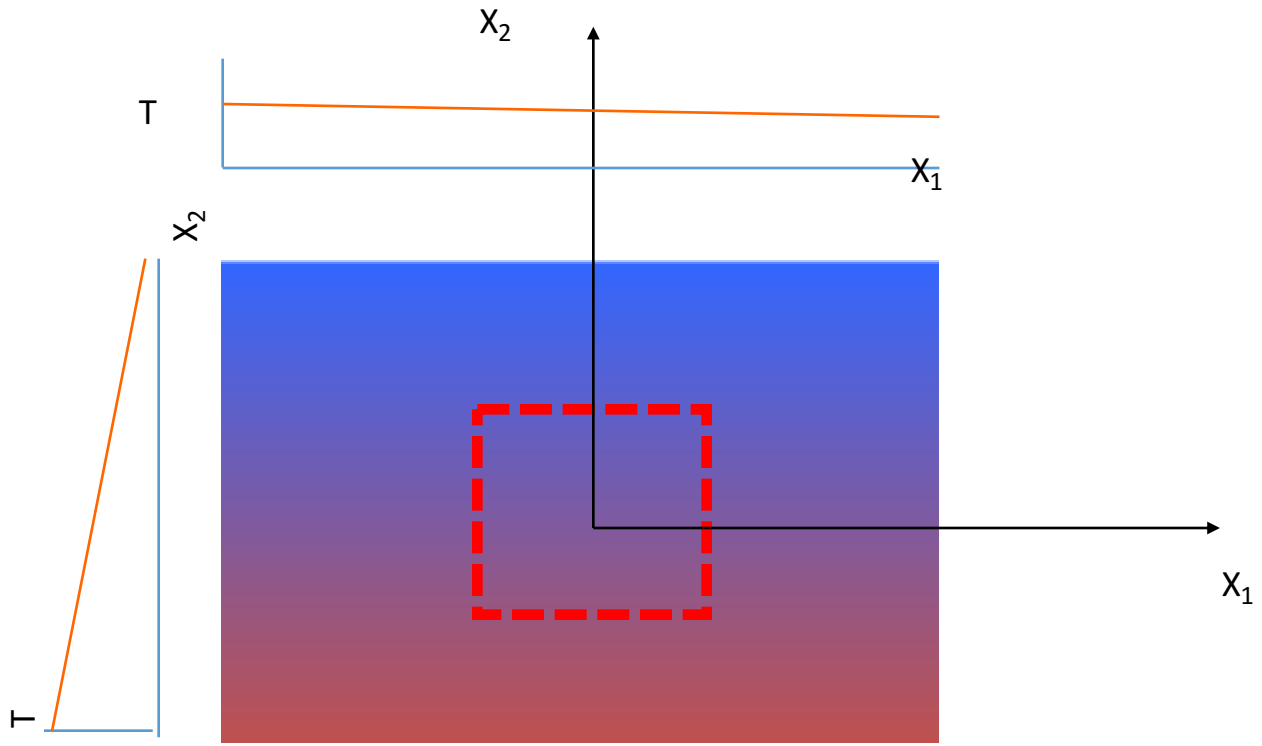

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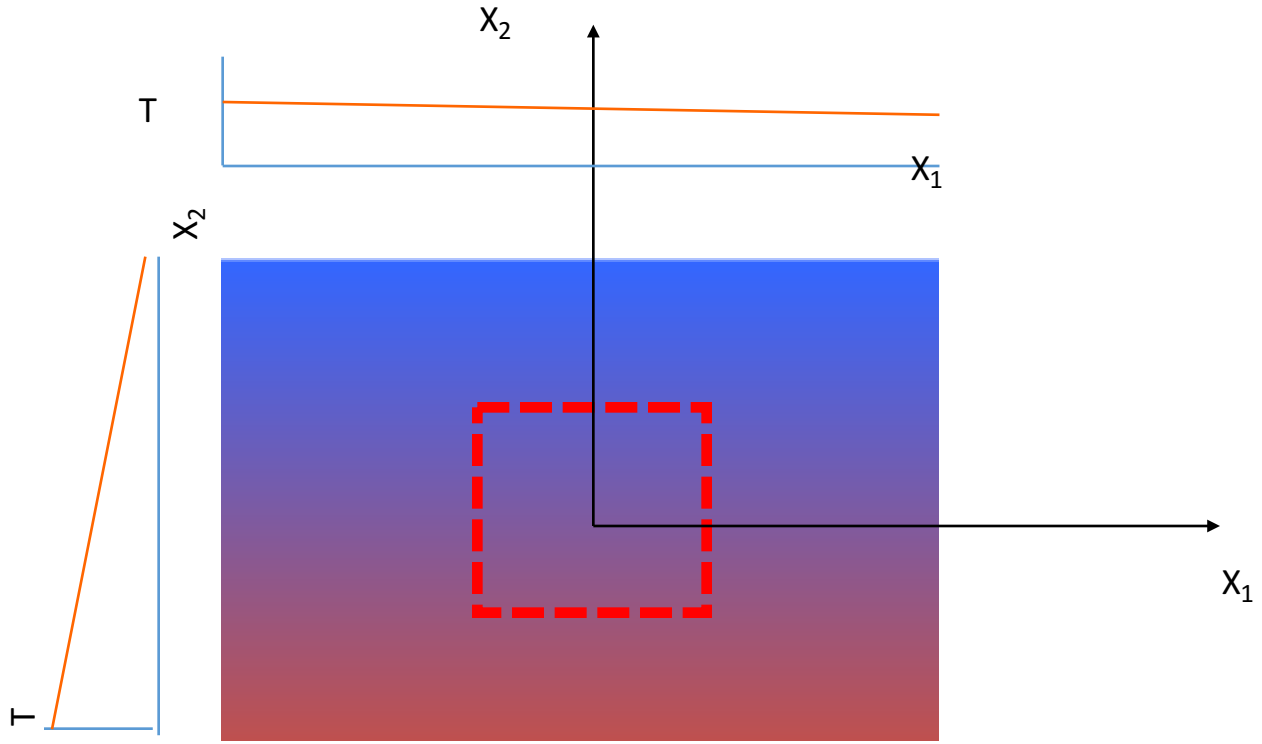
REPEAT? What's happening in the red bounded box?



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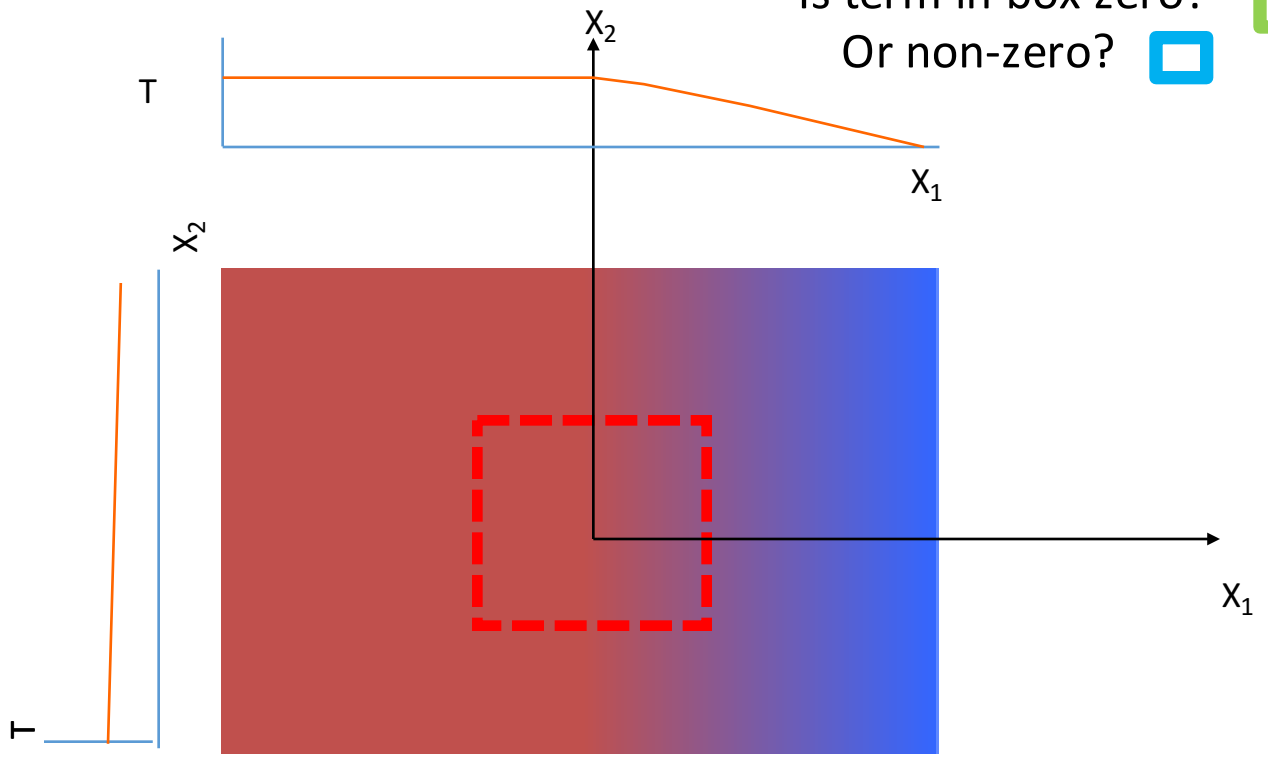
What's happening in the red bounded box?

Is term in box zero?

Or non-zero?



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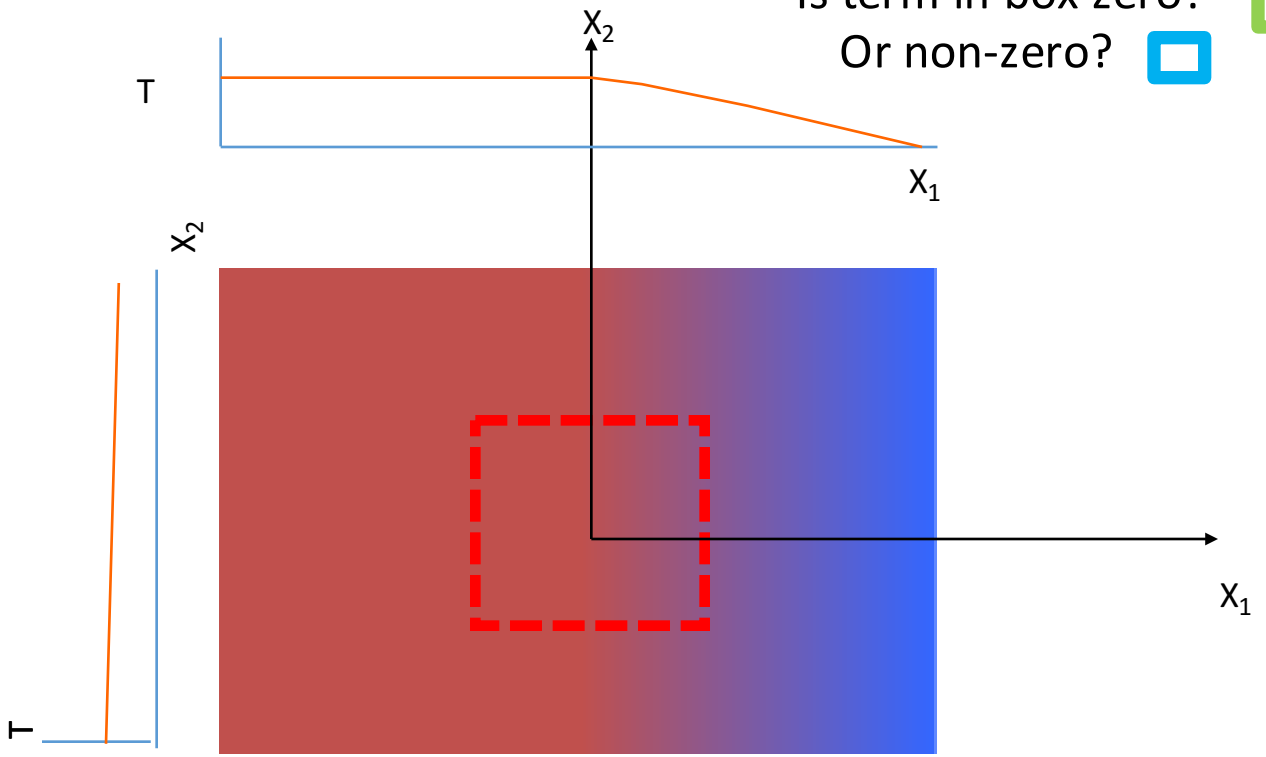

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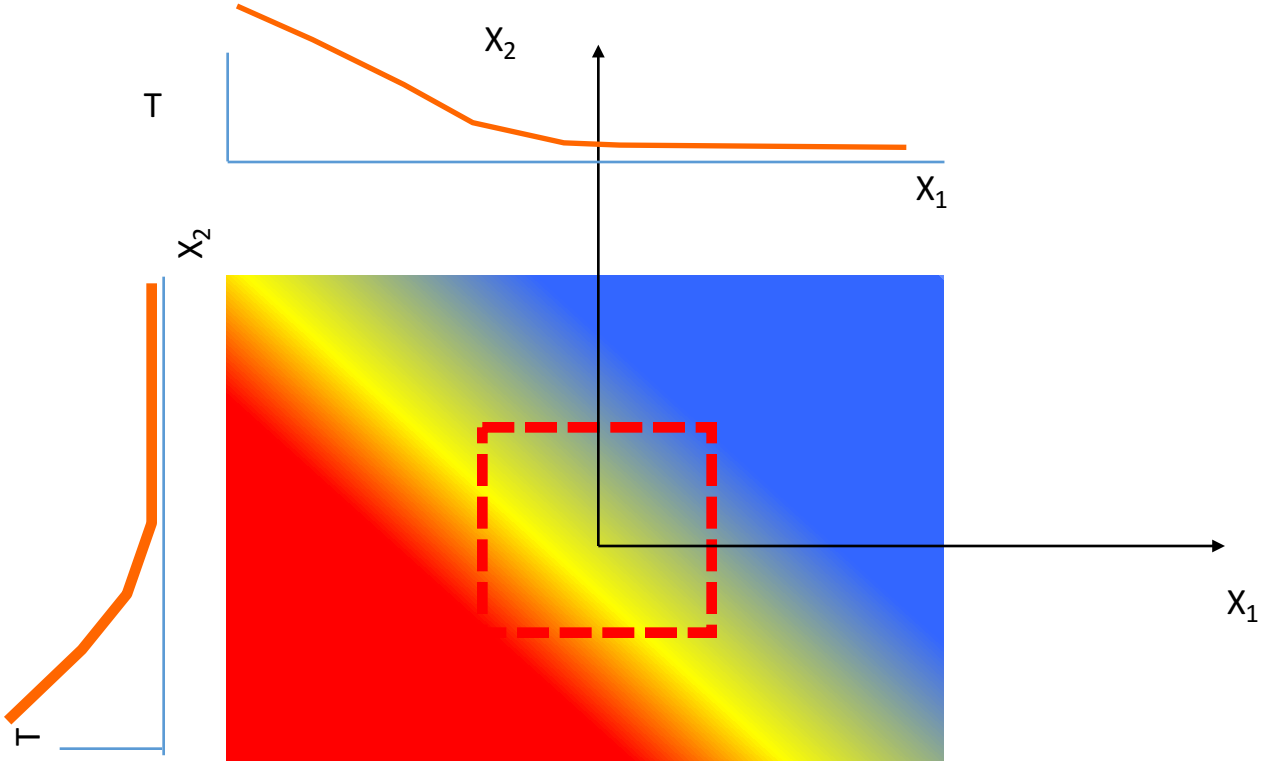

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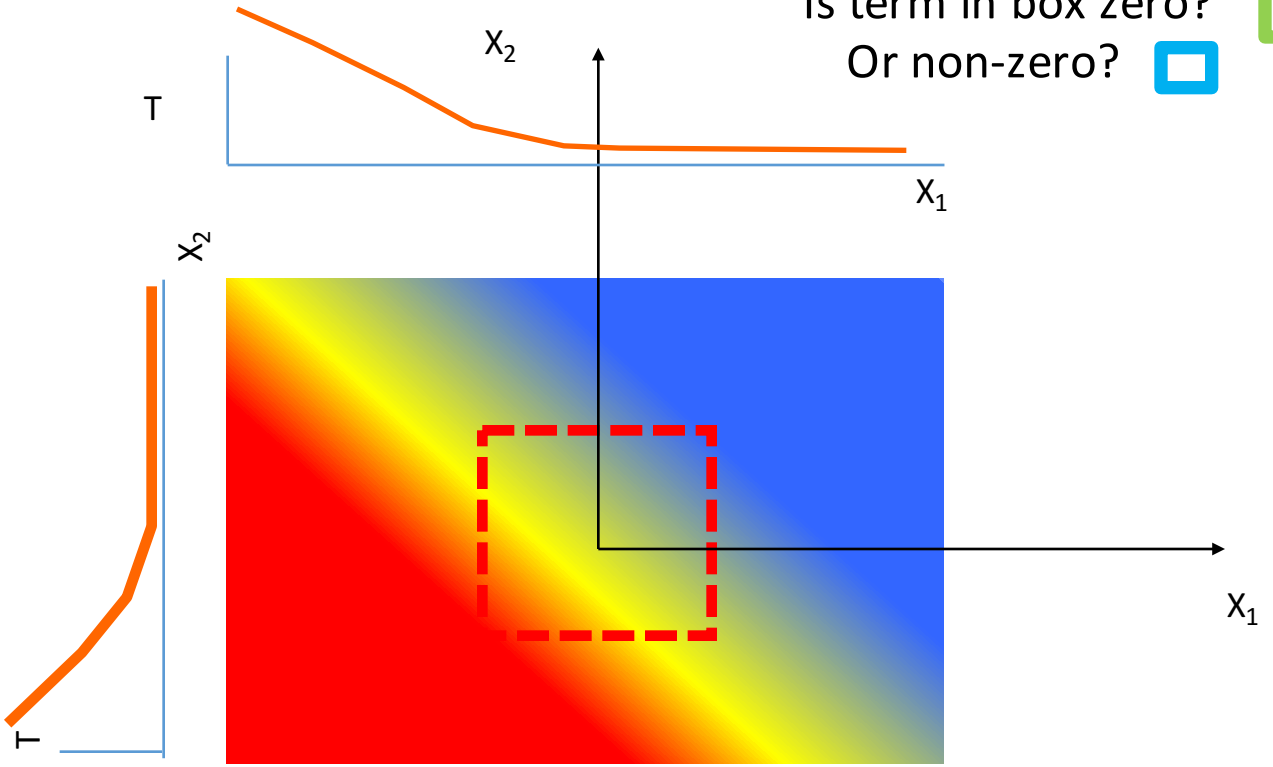

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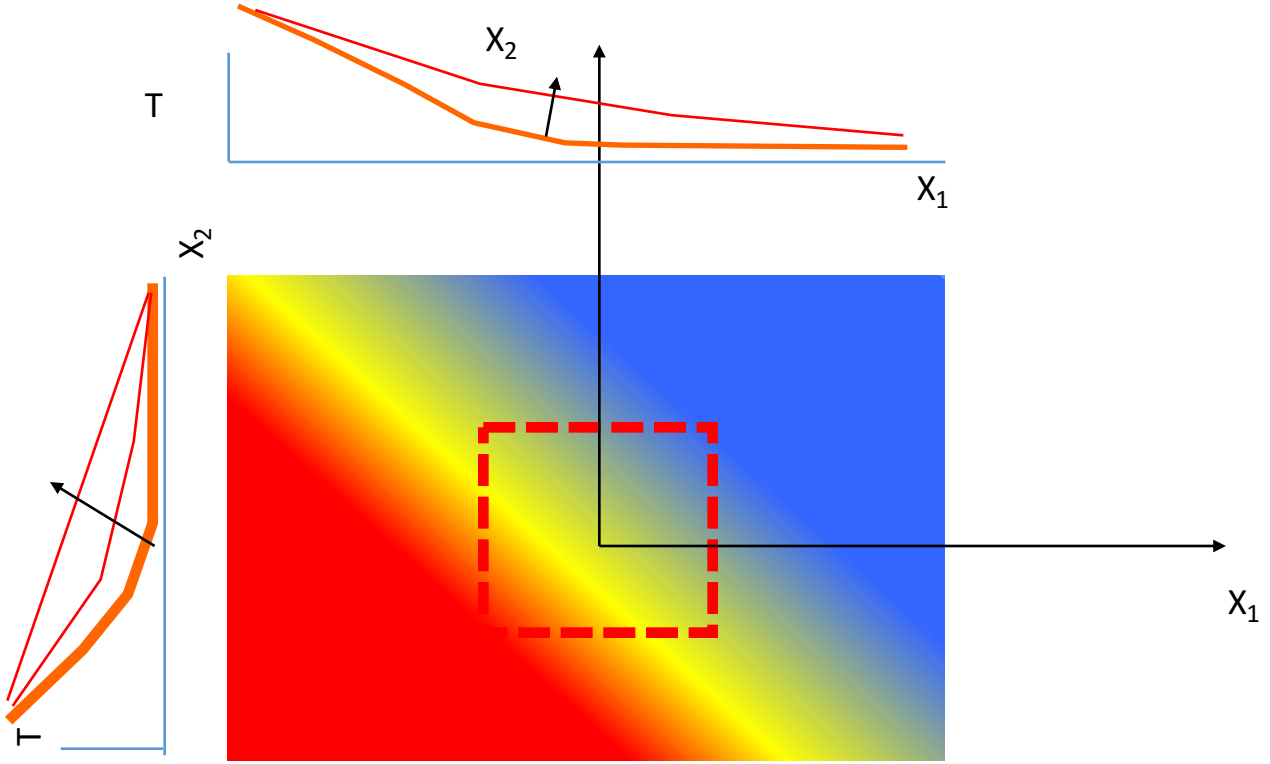
? **What will happen**?

What's happening in the red bounded box?



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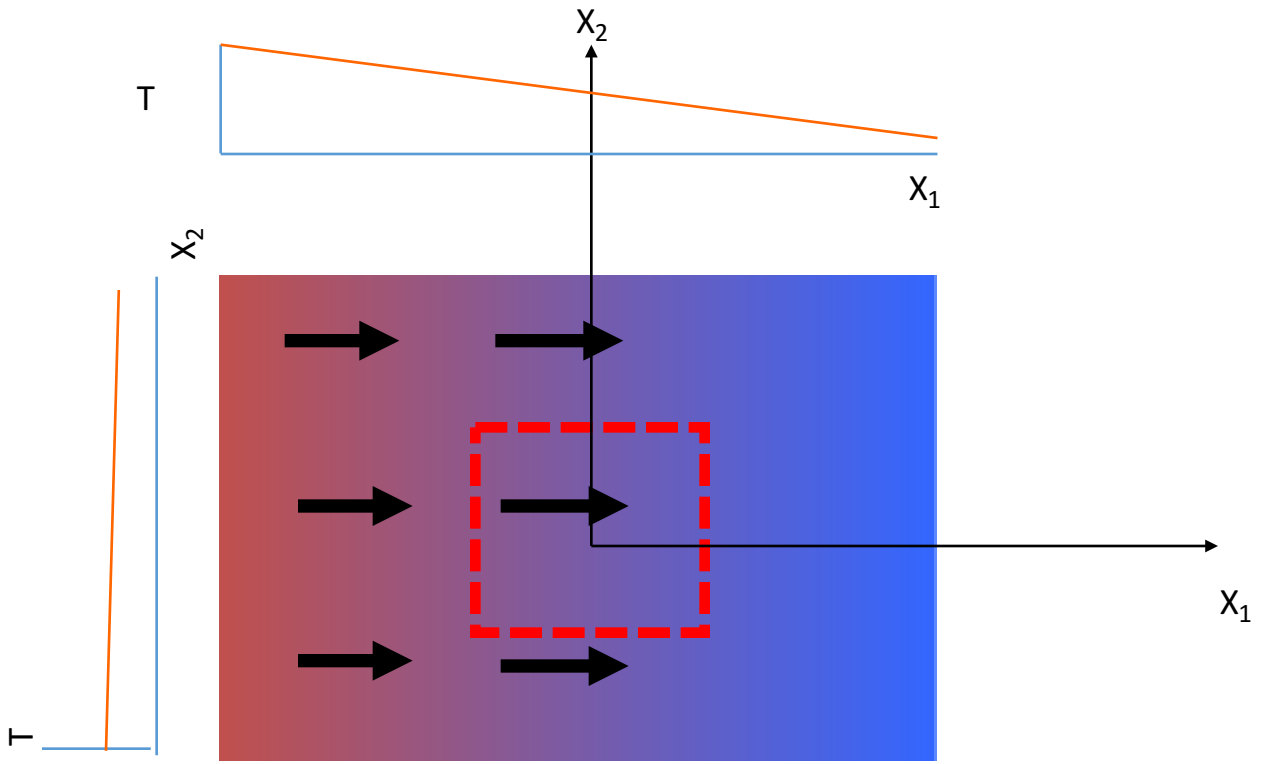
Expect T rise

ADD FLOW



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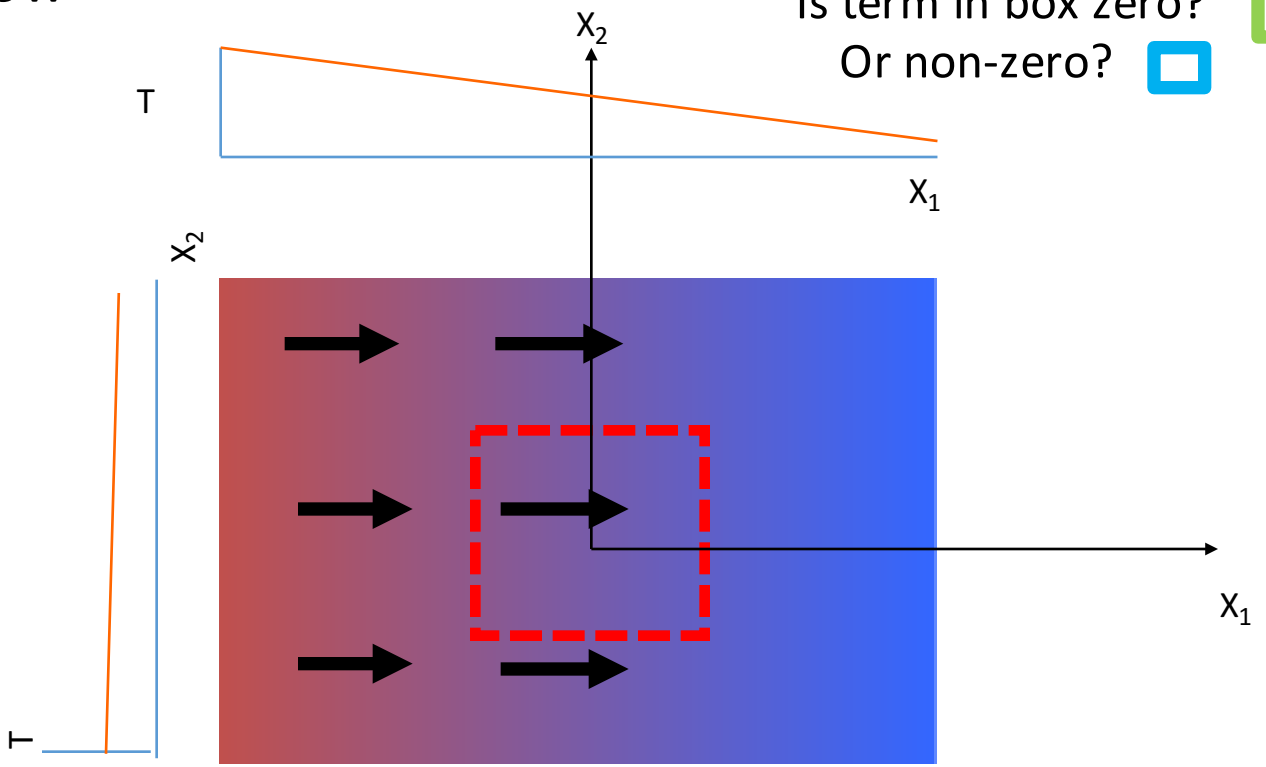
ADD FLOW

Is term in box zero?

Or non-zero?



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ADD FLOW

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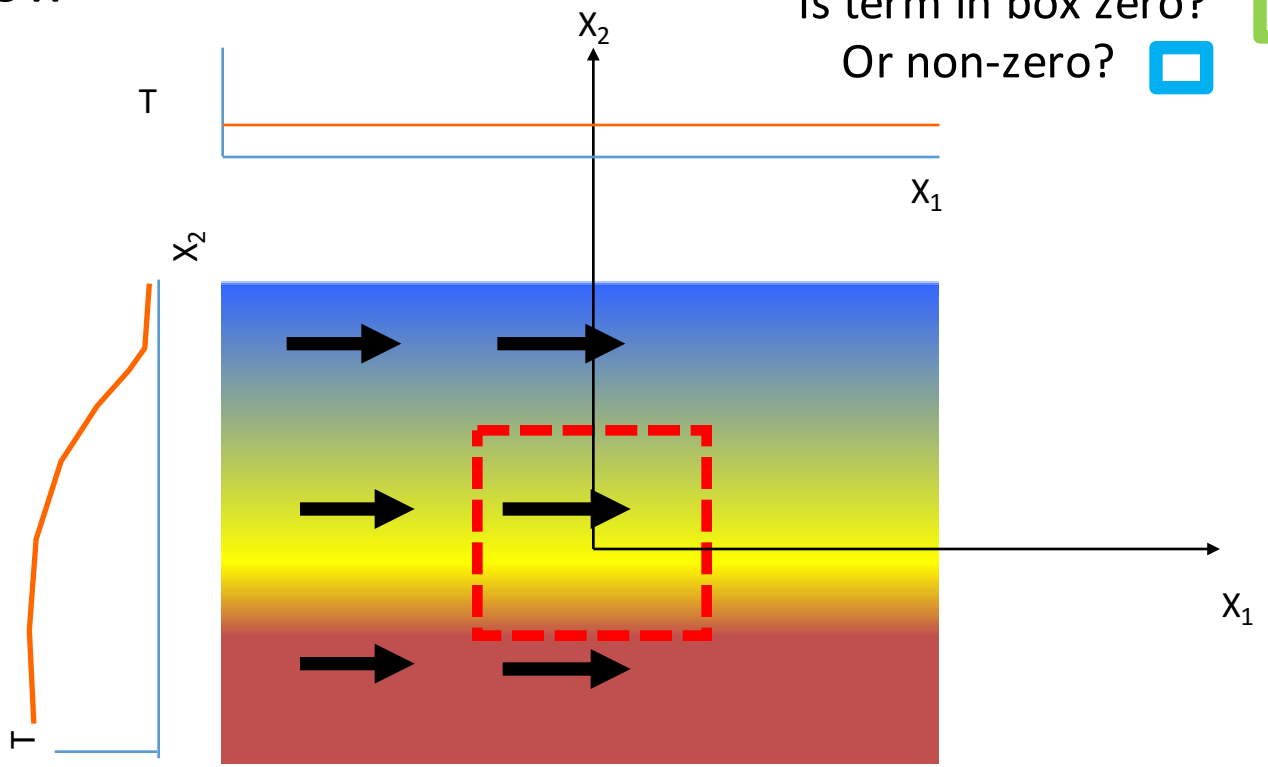

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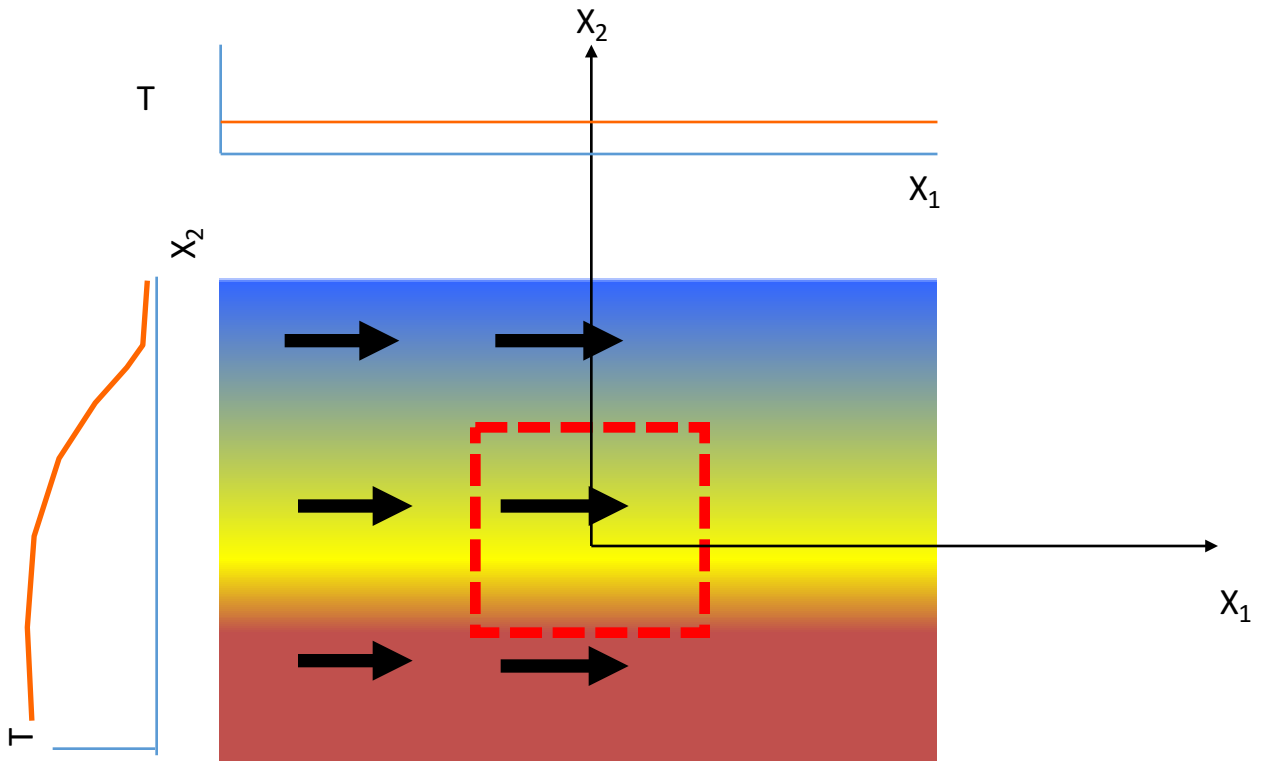
ANSWER



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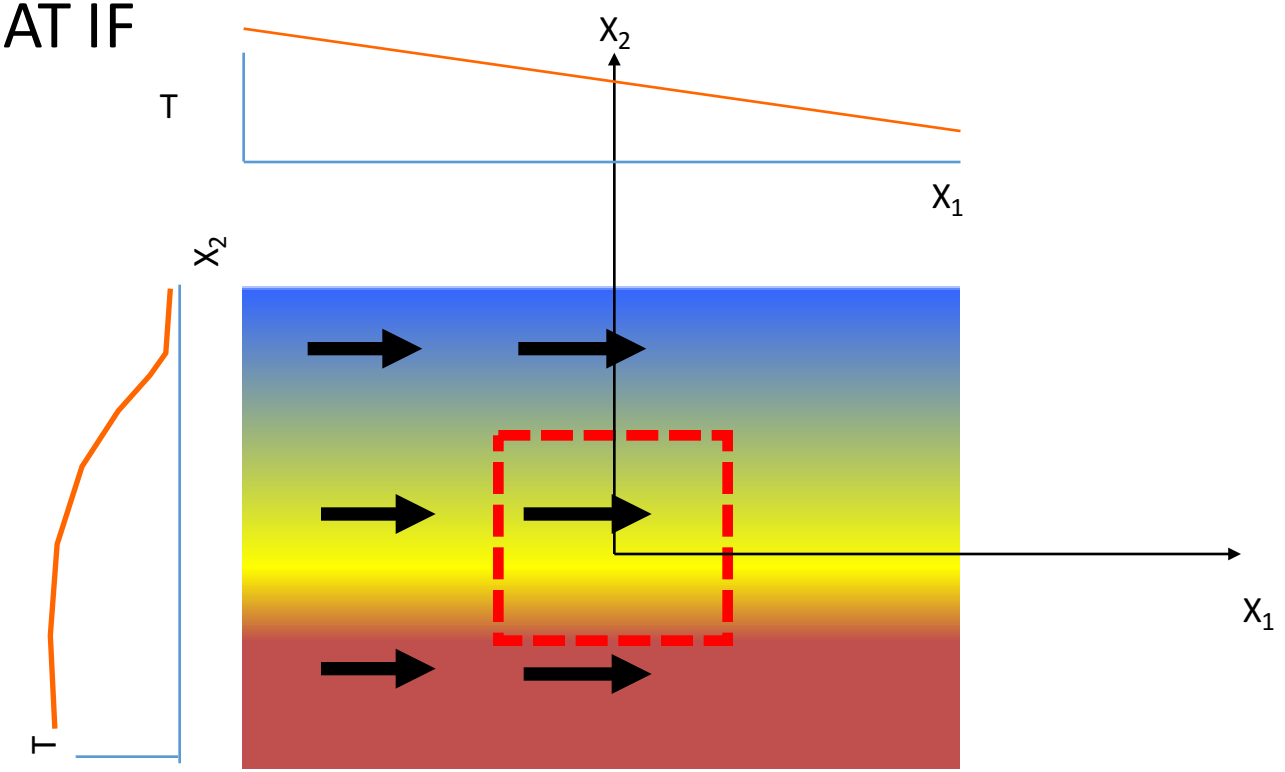
WHAT IF



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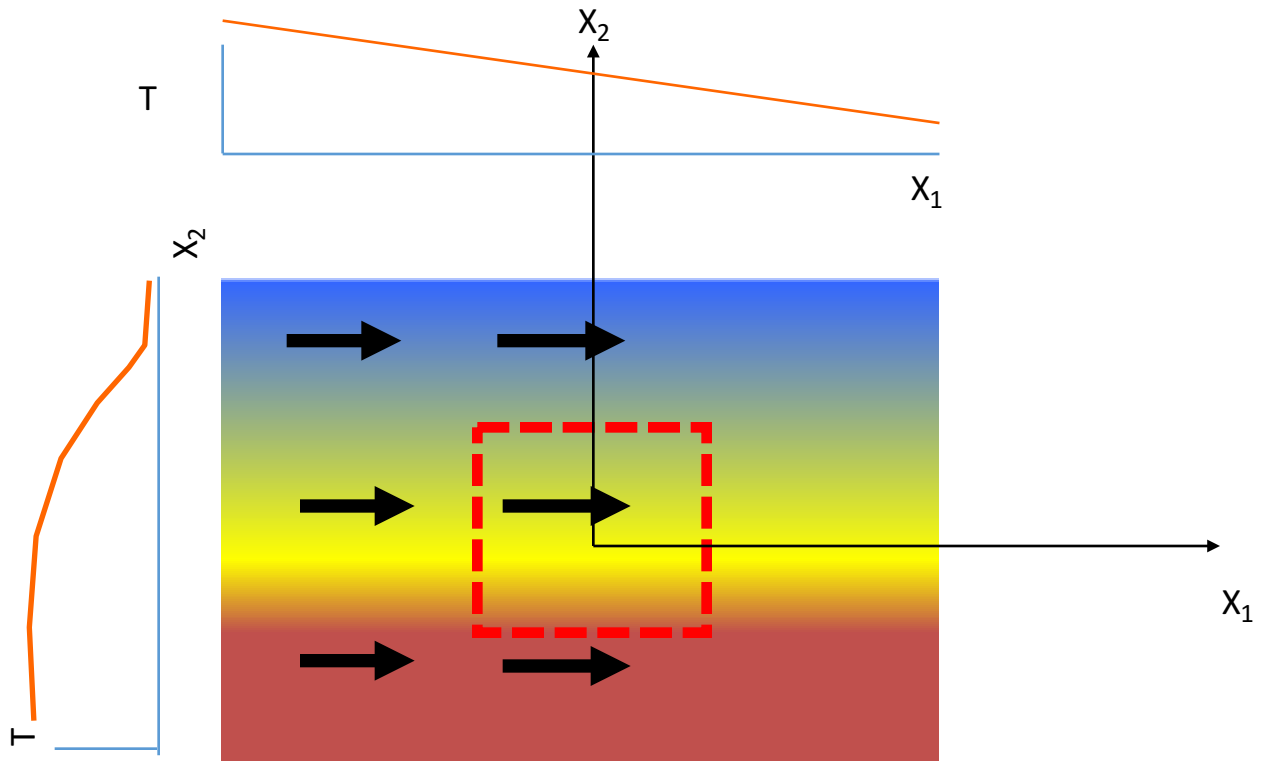
WHAT IF



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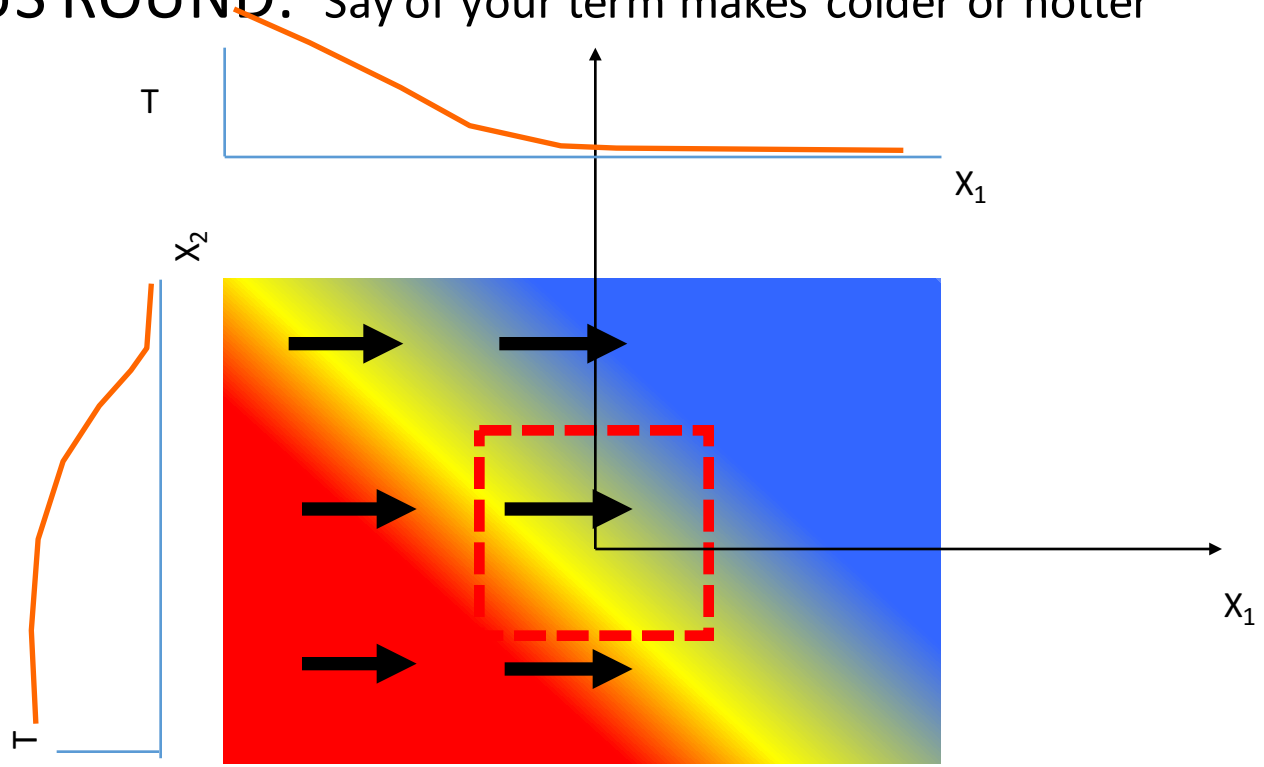
BONUS ROUND: Say of your term makes colder or hotter



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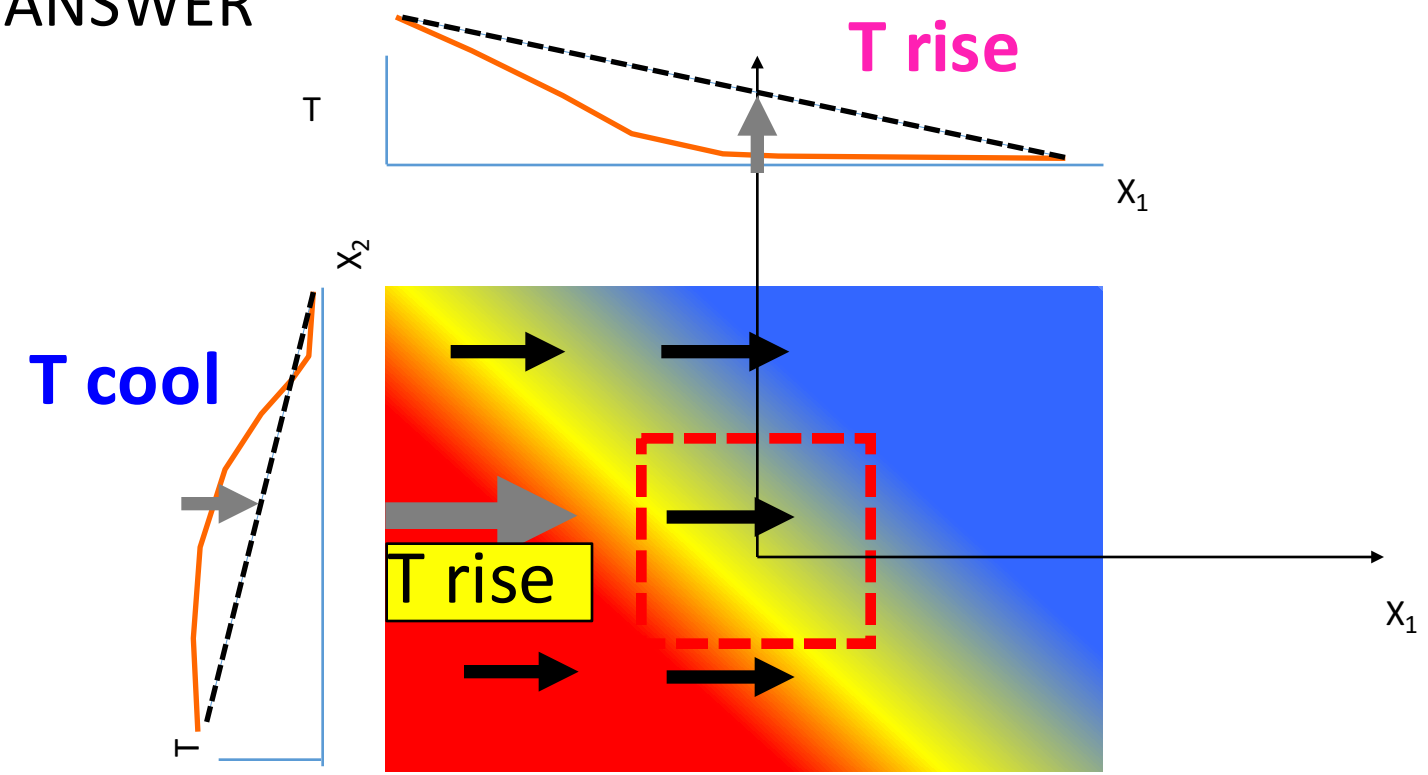
ANSWER



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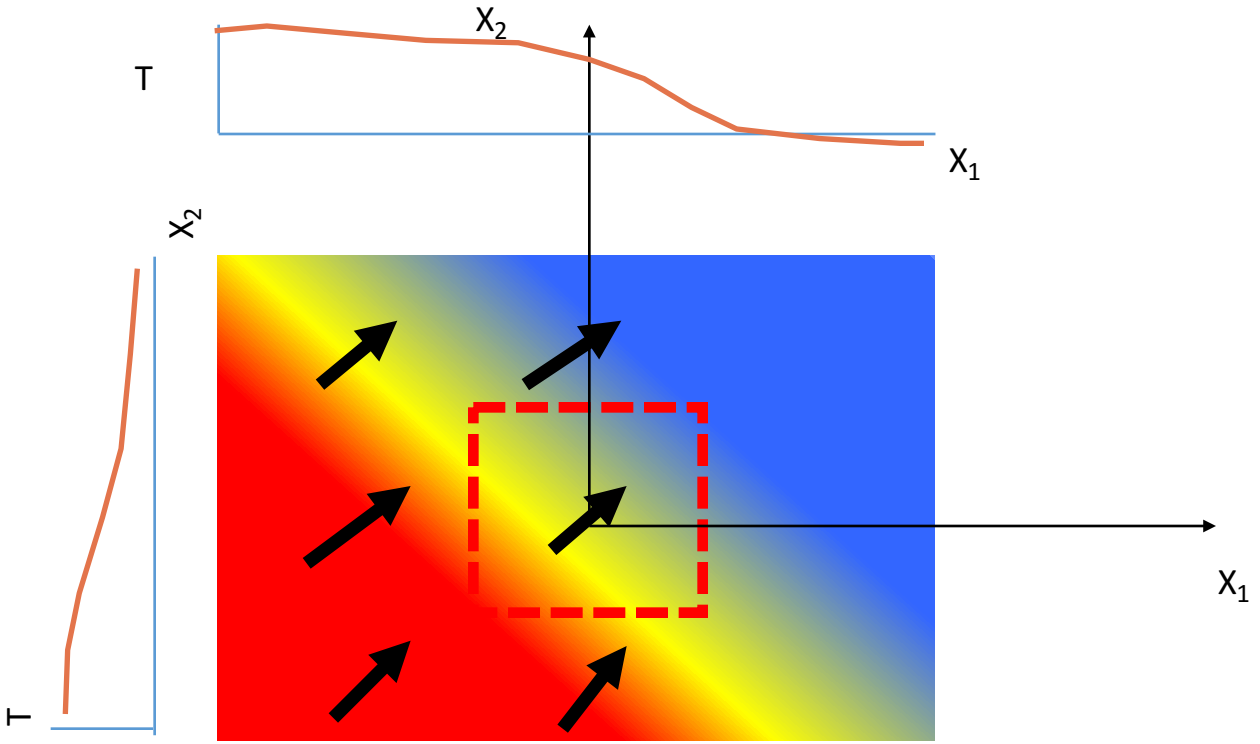
$\partial T/\partial t$ | $u_1 \partial T/\partial x_1$ | $u_2 \partial T/\partial x_2$ | $\frac{\partial}{\partial x_1}(K \partial T/\partial x_1)$ | $\frac{\partial}{\partial x_2}(K \partial T/\partial x_2)$

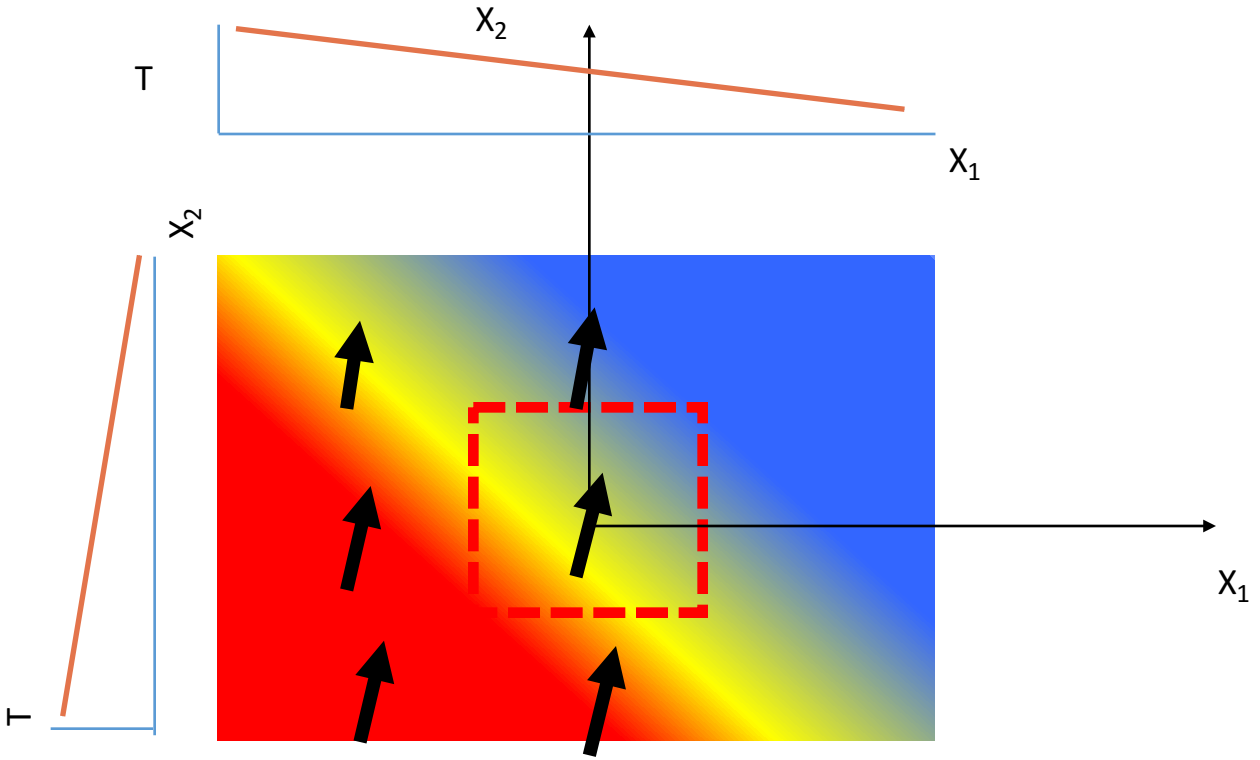
BONUS ROUND



As a group tell me for each term: If your term is ____, then _____
a) zero magnitude (sit), b) medium size (squat), c) big magnitude, (stand)

$\partial T/\partial t$ $\quad u_1\, \partial T/\partial x_1$ $\quad u_2\, \partial T/\partial x_2$ $\quad \frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$ $\quad \frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

BONUS ROUND

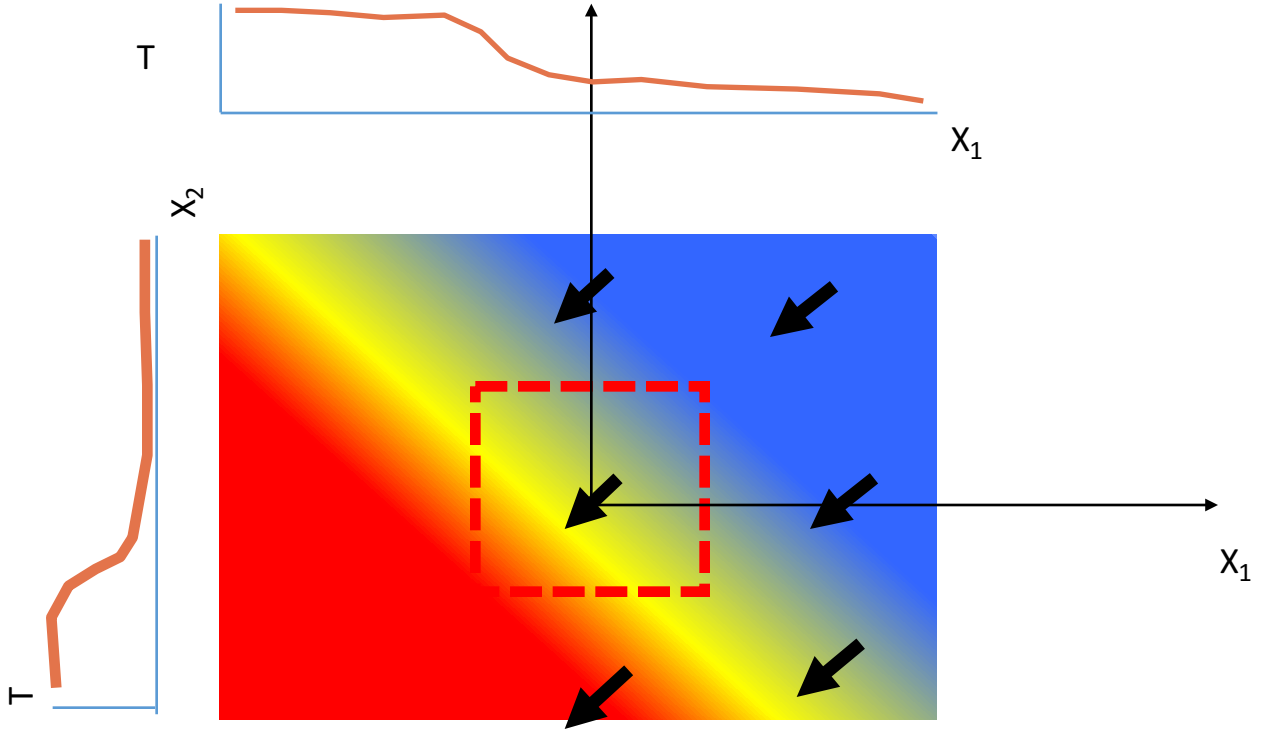


As a group tell me for each term: If your term is ____, then _____
a) zero magnitude (sit), b) medium size (squat), c) big magnitude, (stand)

$\partial T/\partial t$ $\quad u_1\, \partial T/\partial x_1$ $\quad u_2\, \partial T/\partial x_2$ $\quad \frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$ $\quad \frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

For 2015. add case where they need to decide if advection
And diffusion are both adding to temp or detracting.
Make them reason out the sign of the terms.

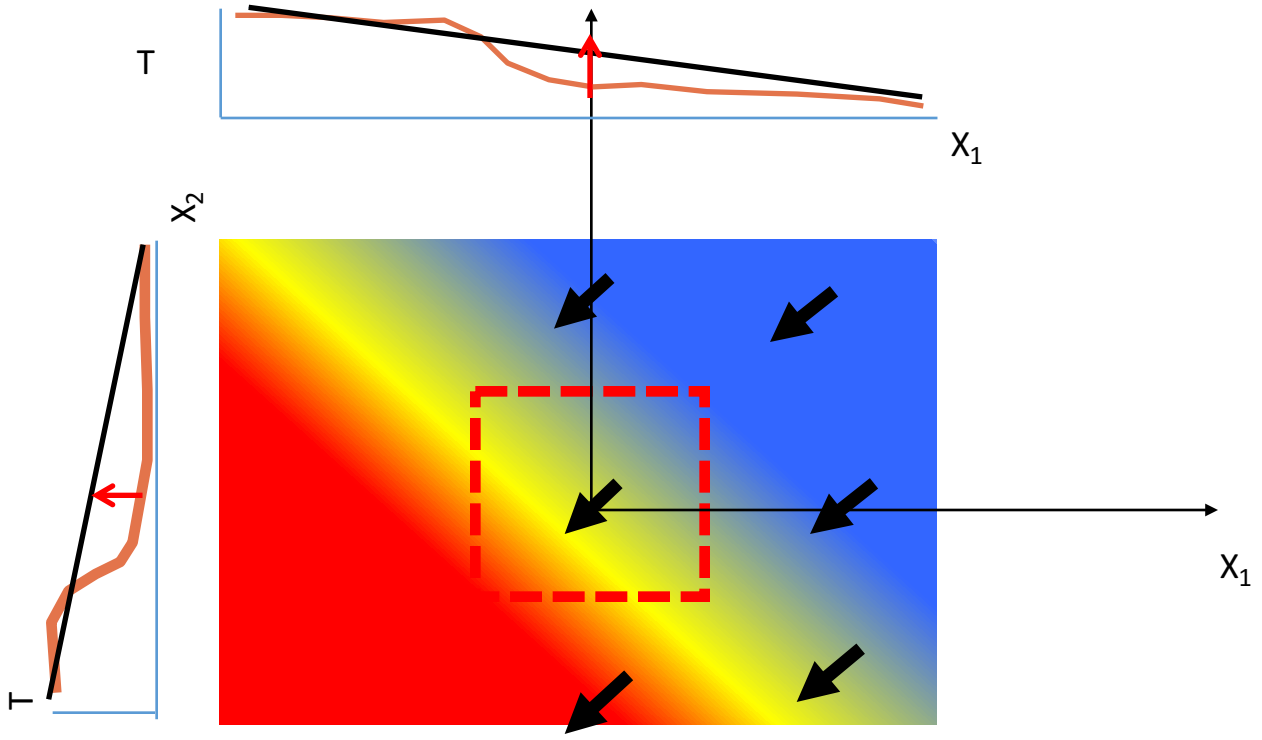
DAILY DOUBLE….SIGNS MATTER!!!!



As a group tell me for each term: If your term is ____, then _____
a) zero magnitude (sit), b) medium size (squat), c) big magnitude, (stand)

$\partial T/\partial t$ $\quad u_1\,\partial T/\partial x_1$ $\quad u_2\,\partial T/\partial x_2$ $\quad \frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$ $\quad \frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$

DAILY DOUBLE…



What is advection trying to do?

What is diffusion trying to do?

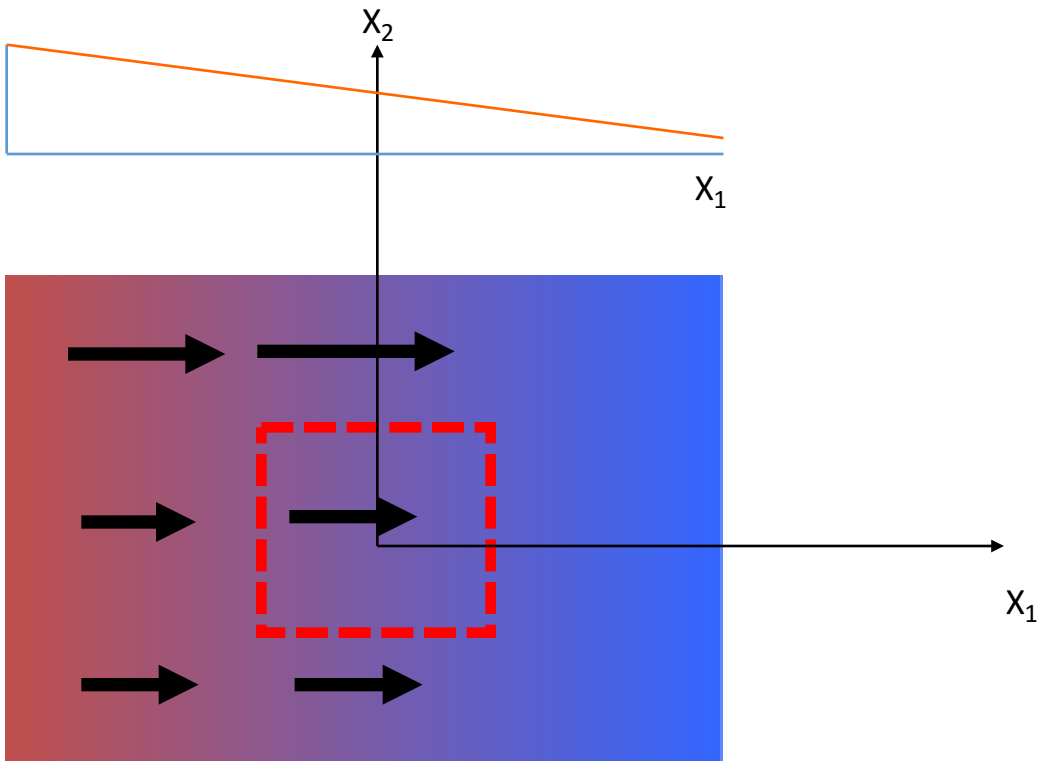
$\partial T/\partial t$ $\quad u_1\, \partial T/\partial x_1$ $\quad u_2\, \partial T/\partial x_2$ $\quad \frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$ $\quad \frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

In-class worksheets
517/351
4/19/19

ENERGY EQUATION
WITH THE WORK TERMS

Group1

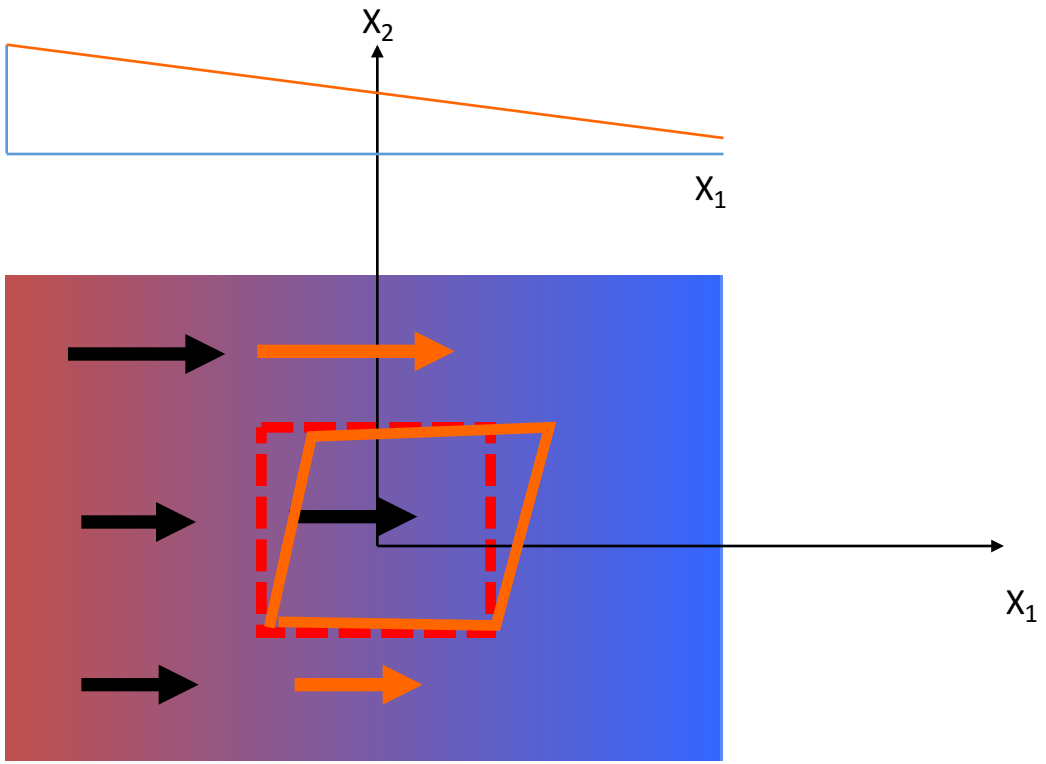
FLOW & Visc Work



$$\partial T/\partial t \quad u_1\,\partial T/\partial x_1 \quad u_2\,\partial T/\partial x_2 \quad \frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1) \quad \frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2) \quad (\partial u_1/\partial x_1)^2 \quad (\partial u_2/\partial x_2)^2 \quad \{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$$

Group1
FLOW & Visc Work

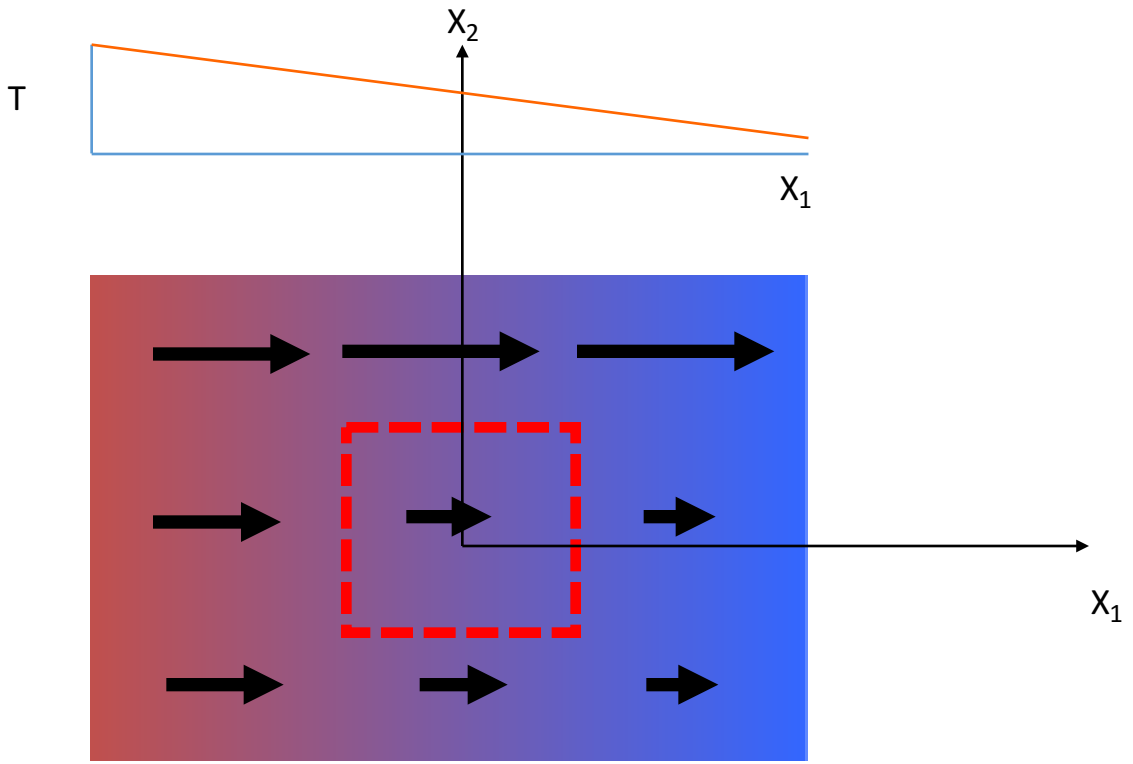
ANSWER



$$\partial T/\partial t \quad \mathbf{u_1\, \partial T/\partial x_1} \quad u_2\, \partial T/\partial x_2 \quad \frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1) \quad \frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2) \quad (\partial u_1/\partial x_1)^2 \quad (\partial u_2/\partial x_2)^2 \quad \{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$$

Group2

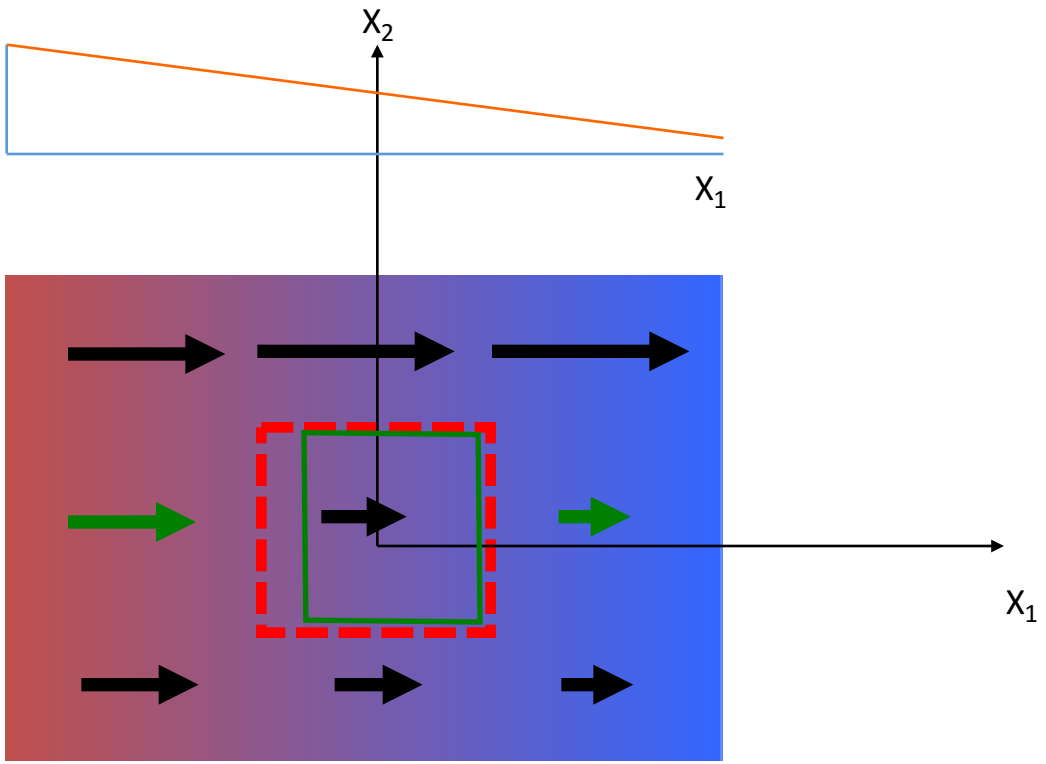
FLOW & Visc Work



$$\partial T/\partial t \quad u_1\,\partial T/\partial x_1 \quad u_2\,\partial T/\partial x_2 \quad \frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1) \quad \frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2) \quad (\partial u_1/\partial x_1)^2 \quad (\partial u_2/\partial x_2)^2 \quad \{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$$

Group2
FLOW & Visc Work

ANSWER



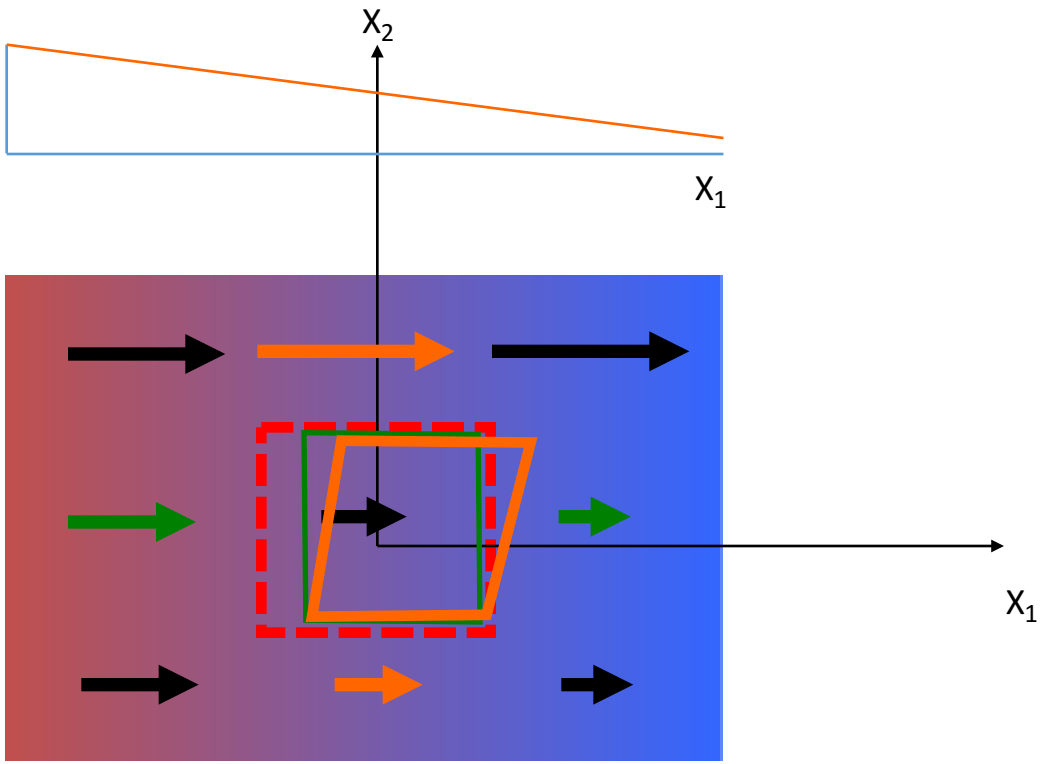
$\partial T/\partial t$ **$u_1\,\partial T/\partial x_1$** $u_2\,\partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$ $\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$

$(\partial u_1/\partial x_1)^2$ $(\partial u_2/\partial x_2)^2$ $\{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$

Group2
FLOW & Visc Work

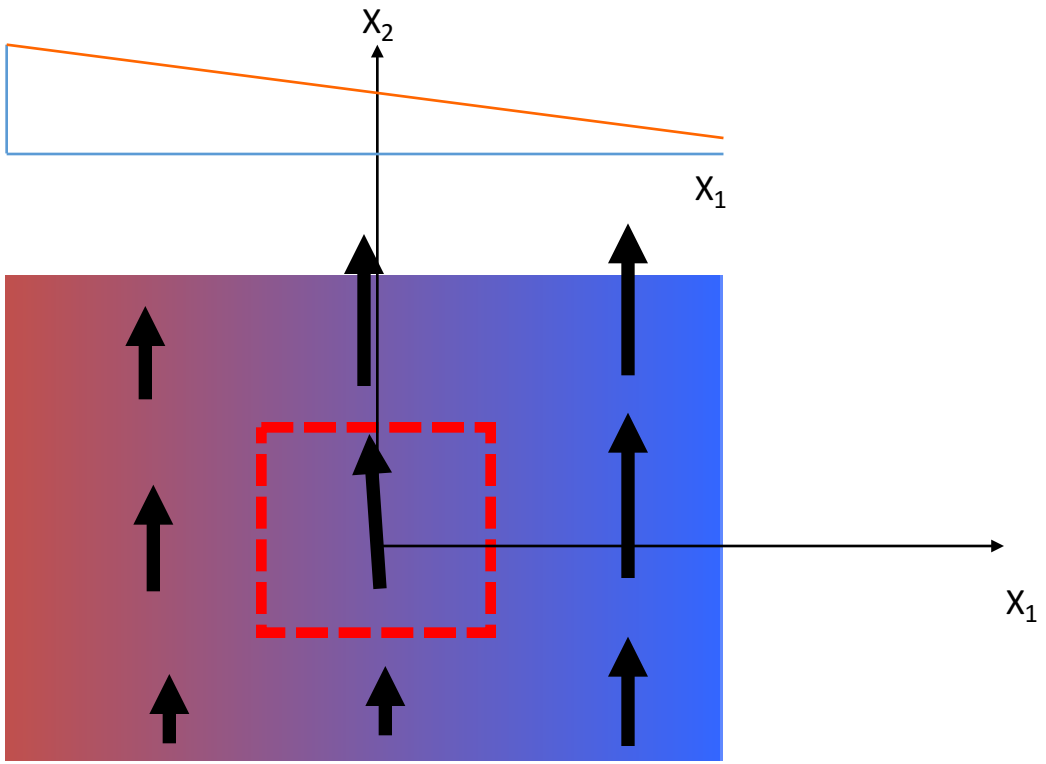
ANSWER



$$\frac{\partial T}{\partial t} \quad u_1 \frac{\partial T}{\partial x_1} \quad u_2 \frac{\partial T}{\partial x_2} \quad \frac{\partial}{\partial x_1}\left(K \frac{\partial T}{\partial x_1}\right) \quad \frac{\partial}{\partial x_2}\left(K \frac{\partial T}{\partial x_2}\right) \quad \left(\frac{\partial u_1}{\partial x_1}\right)^2 \quad \left(\frac{\partial u_2}{\partial x_2}\right)^2 \quad \left\{\left[\frac{\partial u_1}{\partial x_2}\right) + \left(\frac{\partial u_2}{\partial x_1}\right)\right\}^2$$

Group3

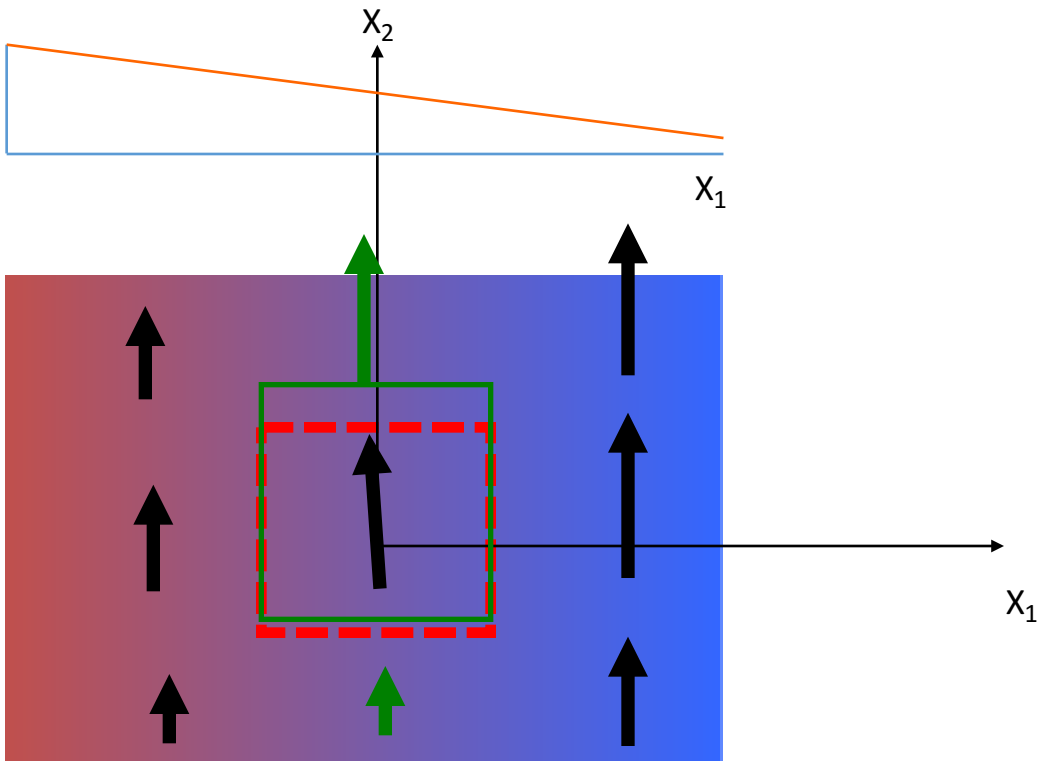
FLOW & Visc Work



$$\frac{\partial T}{\partial t} \quad u_1 \frac{\partial T}{\partial x_1} \quad u_2 \frac{\partial T}{\partial x_2} \quad \frac{\partial}{\partial x_1}\left(K \frac{\partial T}{\partial x_1}\right) \quad \frac{\partial}{\partial x_2}\left(K \frac{\partial T}{\partial x_2}\right) \quad \left(\frac{\partial u_1}{\partial x_1}\right)^2 \quad \left(\frac{\partial u_2}{\partial x_2}\right)^2 \quad \left\{\left[\frac{\partial u_1}{\partial x_2}\right) + \left(\frac{\partial u_2}{\partial x_1}\right)\right\}^2$$

Group3
FLOW & Visc Work

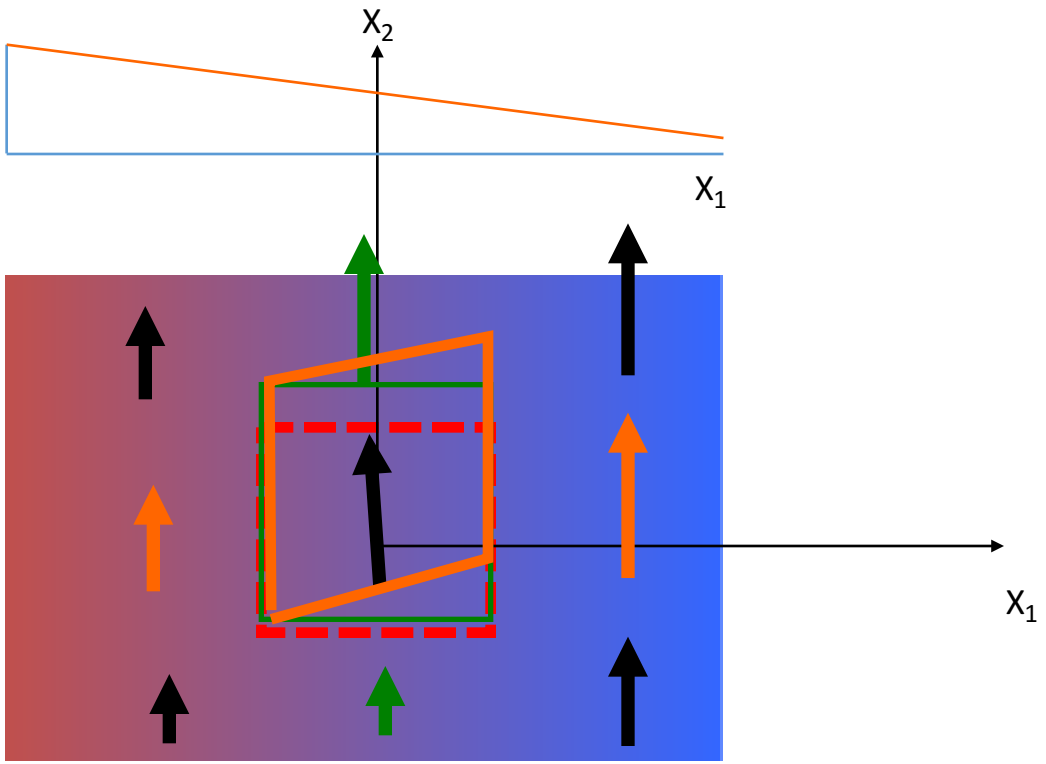
ANSWER



$$\frac{\partial T}{\partial t} = u_1 \frac{\partial T}{\partial x_1} + u_2 \frac{\partial T}{\partial x_2} + \frac{\partial}{\partial x_1}\left(K \frac{\partial T}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(K \frac{\partial T}{\partial x_2}\right) + \left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + \left\{\left[\frac{\partial u_1}{\partial x_2}\right) + \left(\frac{\partial u_2}{\partial x_1}\right)\right\}^2$$

Group3
FLOW & Visc Work

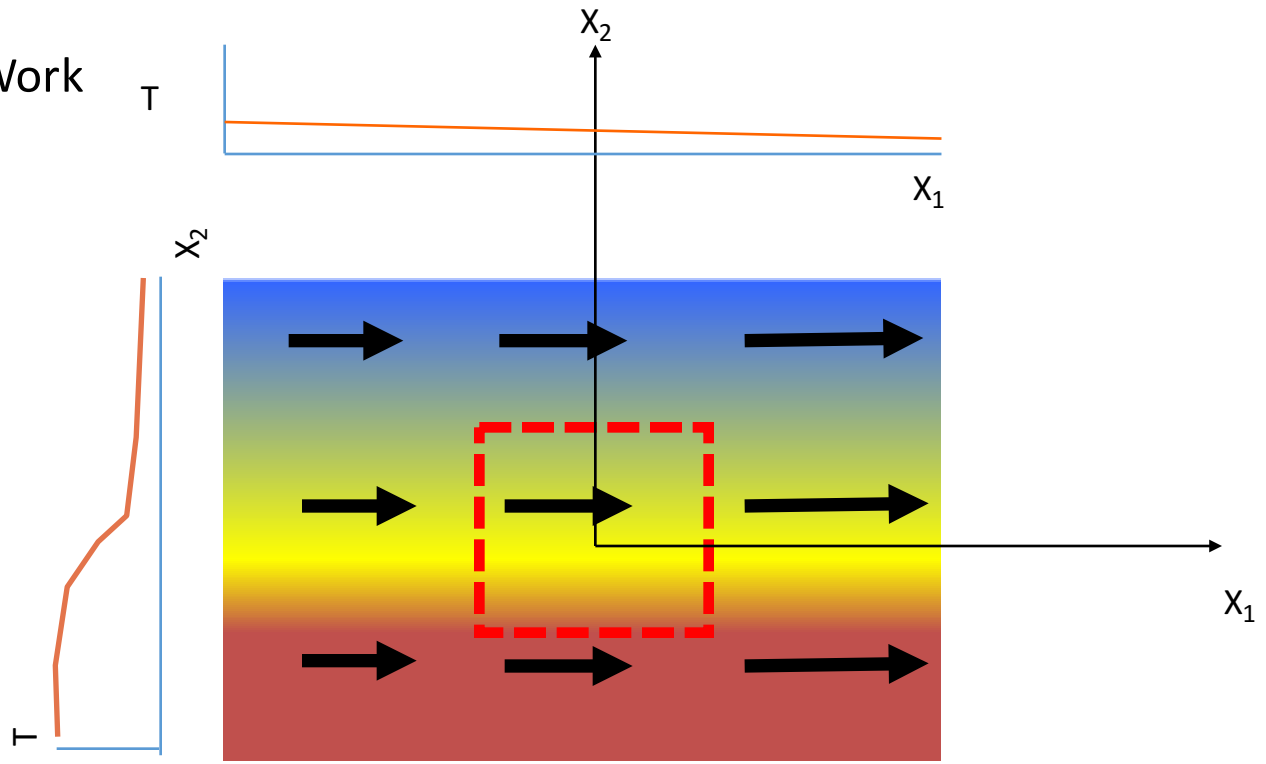
ANSWER



$$\frac{\partial T}{\partial t} = u_1 \frac{\partial T}{\partial x_1} + u_2 \frac{\partial T}{\partial x_2} + \frac{\partial}{\partial x_1}\left(K \frac{\partial T}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(K \frac{\partial T}{\partial x_2}\right) + (\partial u_1/\partial x_1)^2 + (\partial u_2/\partial x_2)^2 + \{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$$

Group4

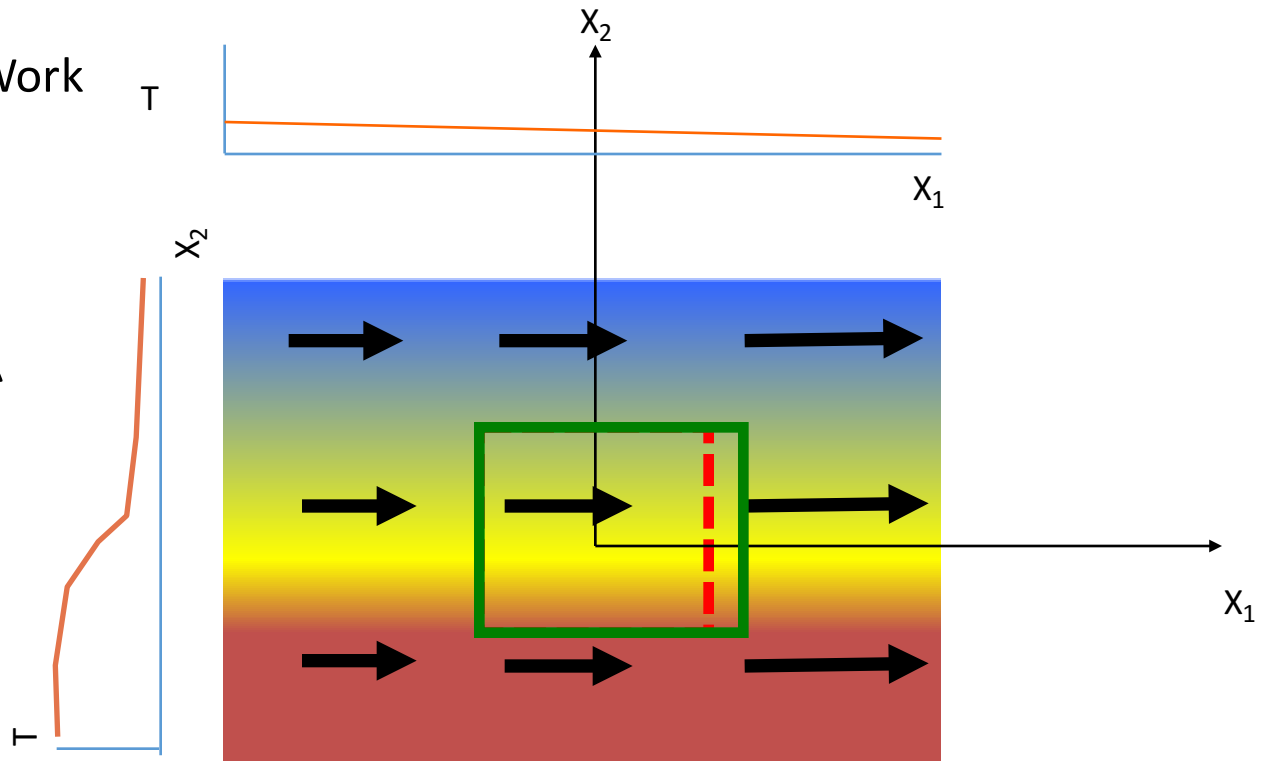
FLOW & Visc Work



$\partial T/\partial t$ $u_1\ \partial T/\partial x_1$ $u_2\ \partial T/\partial x_2$ $\frac{\partial}{\partial x_1}(K\ \partial T/\partial x_1)$ $\frac{\partial}{\partial x_2}(K\ \partial T/\partial x_2)$ $(\partial u_1/\partial x_1)^2$ $(\partial u_2/\partial x_2)^2$ $\{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$

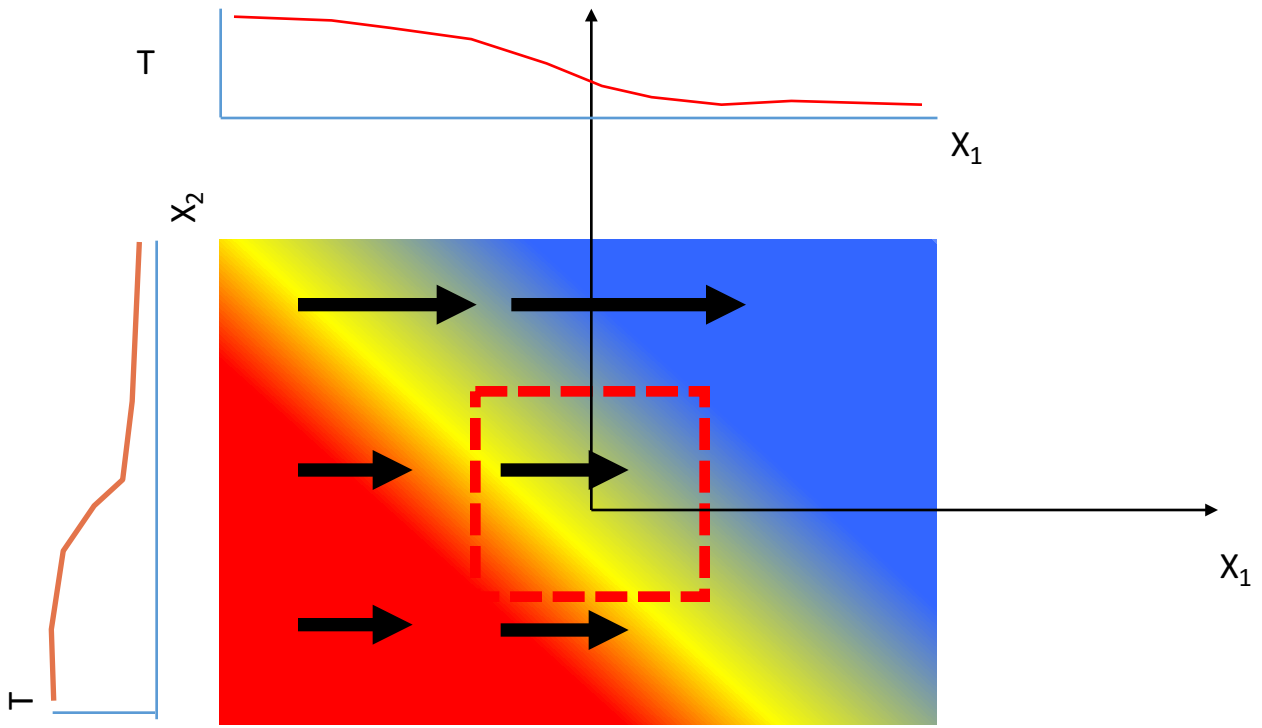
Group4
FLOW & Visc Work

ANSWER



$\partial T/\partial t$ $u_1\ \partial T/\partial x_1$ $u_2\ \partial T/\partial x_2$ $\frac{\partial}{\partial x_1}(K\ \partial T/\partial x_1)$ **$\frac{\partial}{\partial x_2}(K\ \partial T/\partial x_2)$** **$(\partial u_1/\partial x_1)^2$** $(\partial u_2/\partial x_2)^2$ $\{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$

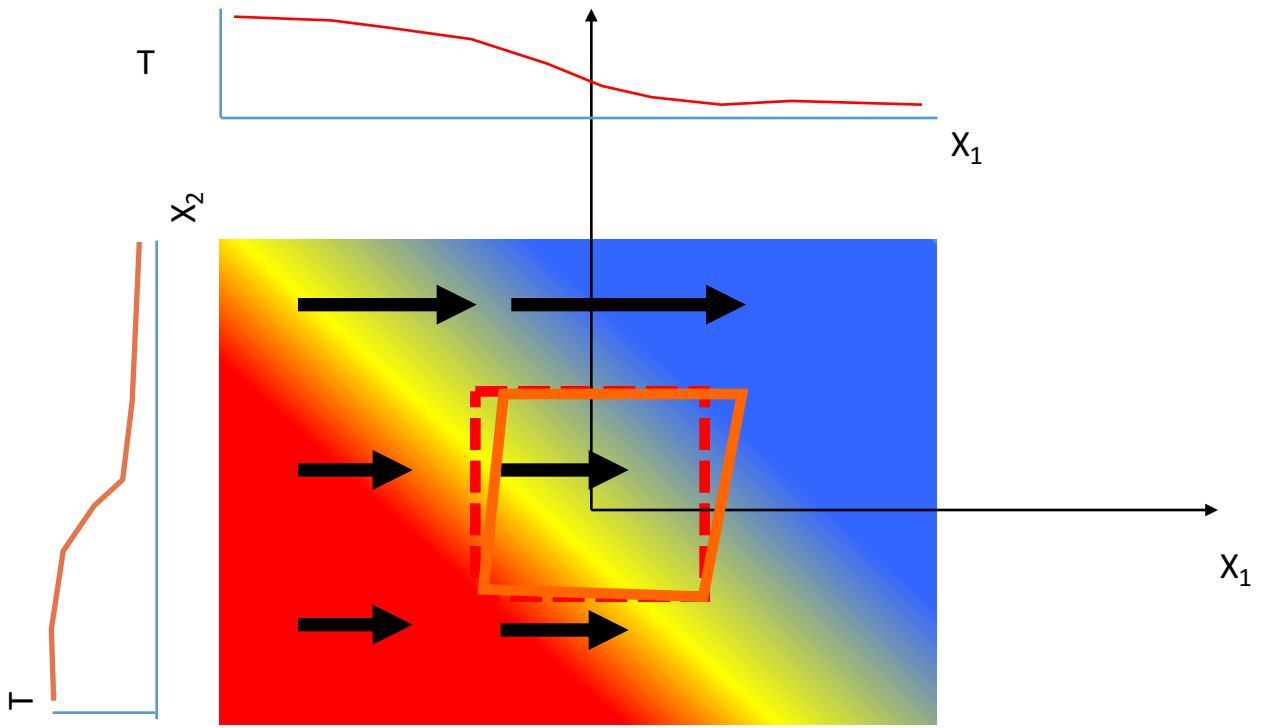
BONUS ROUND 1



$\partial T/\partial t$ $u_1\ \partial T/\partial x_1$ $u_2\ \partial T/\partial x_2$ $\frac{\partial}{\partial x_1}(K\ \partial T/\partial x_1)$ $\frac{\partial}{\partial x_2}(K\ \partial T/\partial x_2)$ $(\partial u_1/\partial x_1)^2$ $(\partial u_2/\partial x_2)^2$ $\{[\partial u_1/\partial x_2) + (\partial u_2/\partial x_1)\}^2$

BONUS
ROUND 1

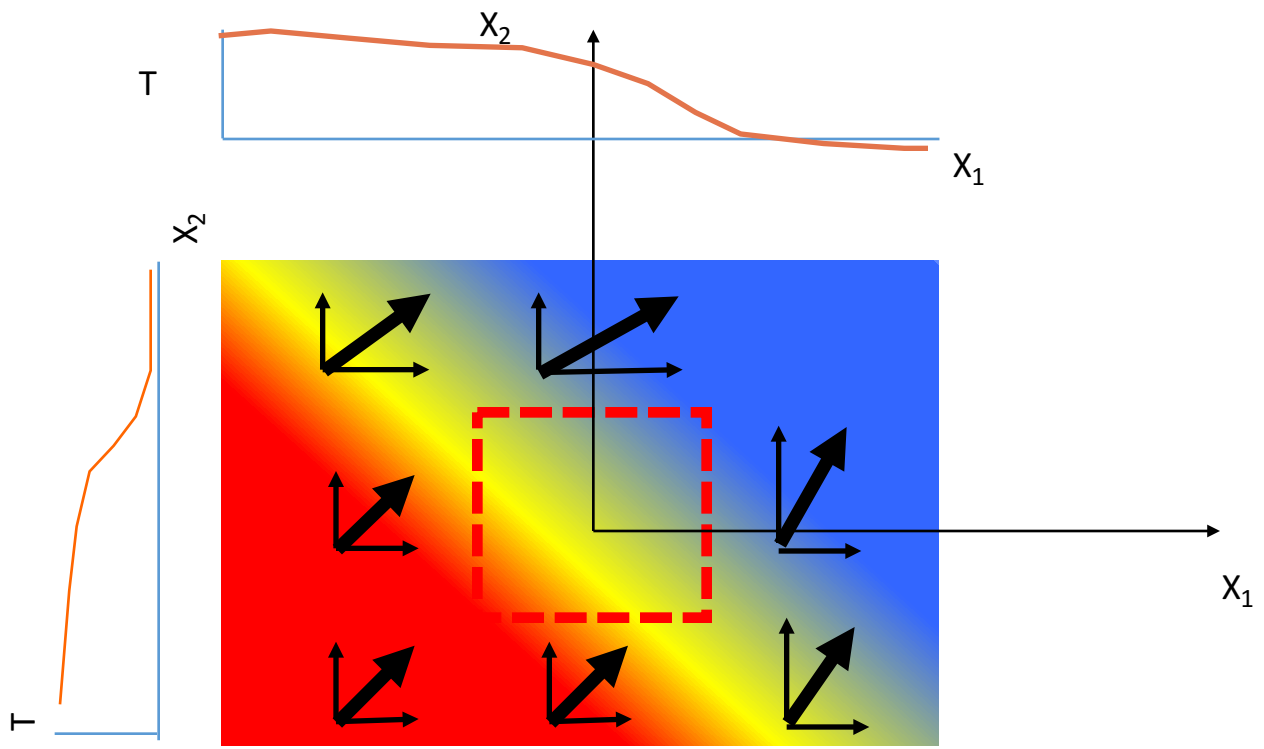
ANSWER



$$\partial T/\partial t \quad \mathbf{u_1\, \partial T/\partial x_1} \quad u_2\, \partial T/\partial x_2 \quad \mathbf{\frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)} \quad \mathbf{\frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)}$$

$$(\partial u_1/\partial x_1)^2 \quad (\partial u_2/\partial x_2)^2 \quad \{[\partial u_1/\partial x_2) + (\mathbf{\partial u_2/\partial x_1})\}^2$$

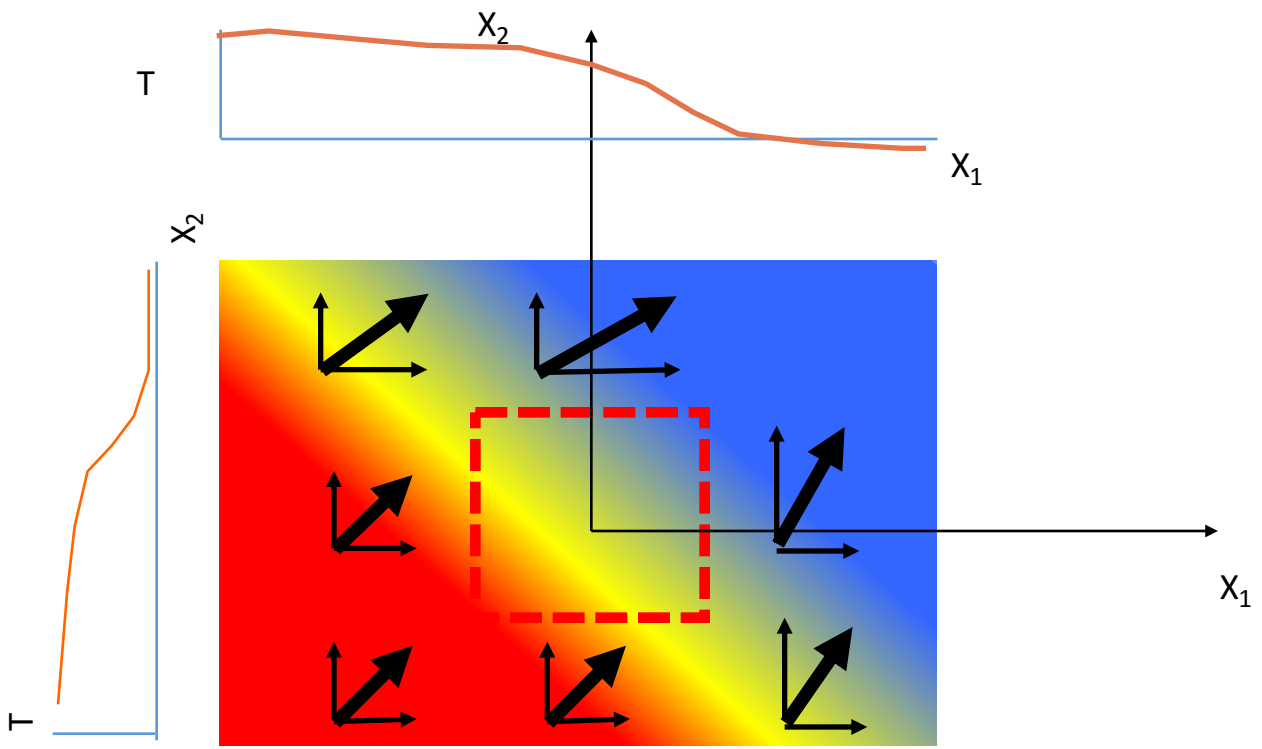
BONUS ROUND 2



$\partial T/\partial t$ $\quad u_1\,\partial T/\partial x_1$ $\quad u_2\,\partial T/\partial x_2$ $\quad \frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$ $\quad \frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$ $\quad (\partial u_1/\partial x_1)^2$ $\quad (\partial u_2/\partial x_2)^2$ $\quad [\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

BONUS
ROUND 2

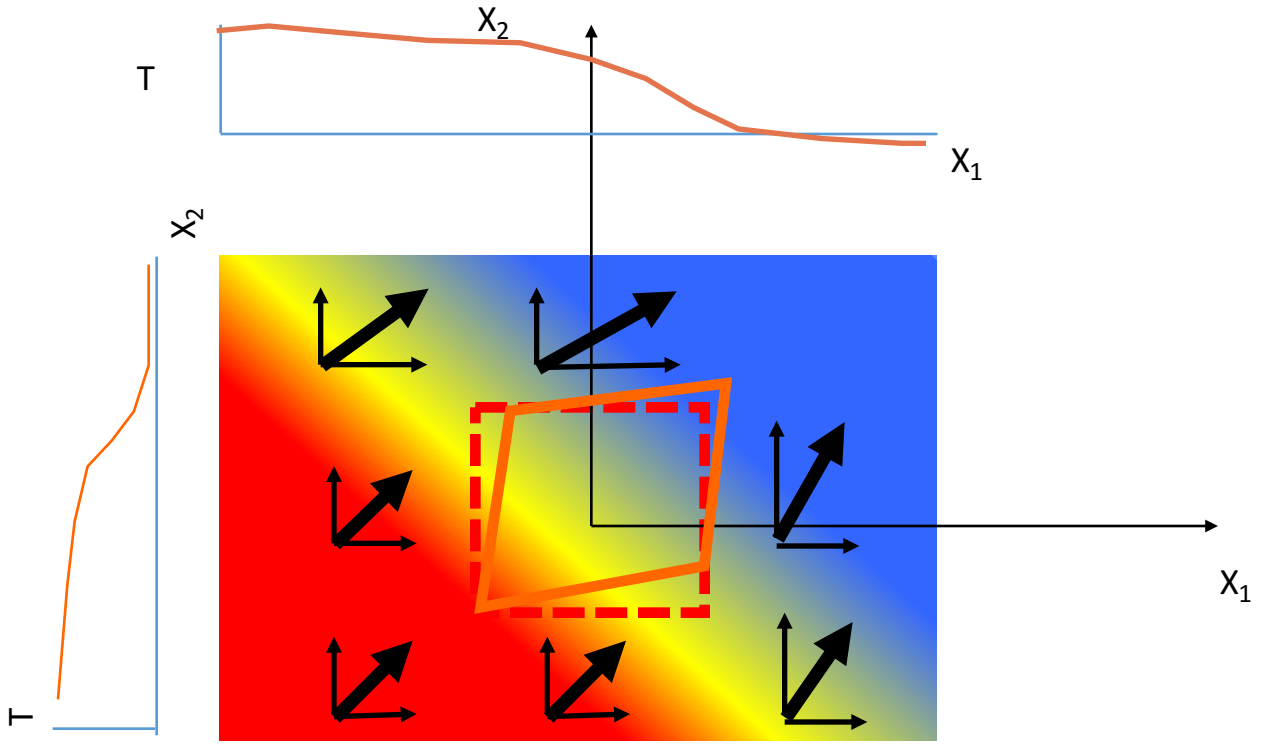
ANSWER



$\partial T/\partial t$ $\quad$ $\mathbf{u_1\,\partial T/\partial x_1}$ $\quad$ $\mathbf{u_2\,\partial T/\partial x_2}$ $\quad$ $\mathbf{\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)}$ $\quad$ $\mathbf{\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)}$ $\quad$ $(\partial u_1/\partial x_1)^2$ $\quad$ $(\partial u_2/\partial x_2)^2$ $\quad$ $[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

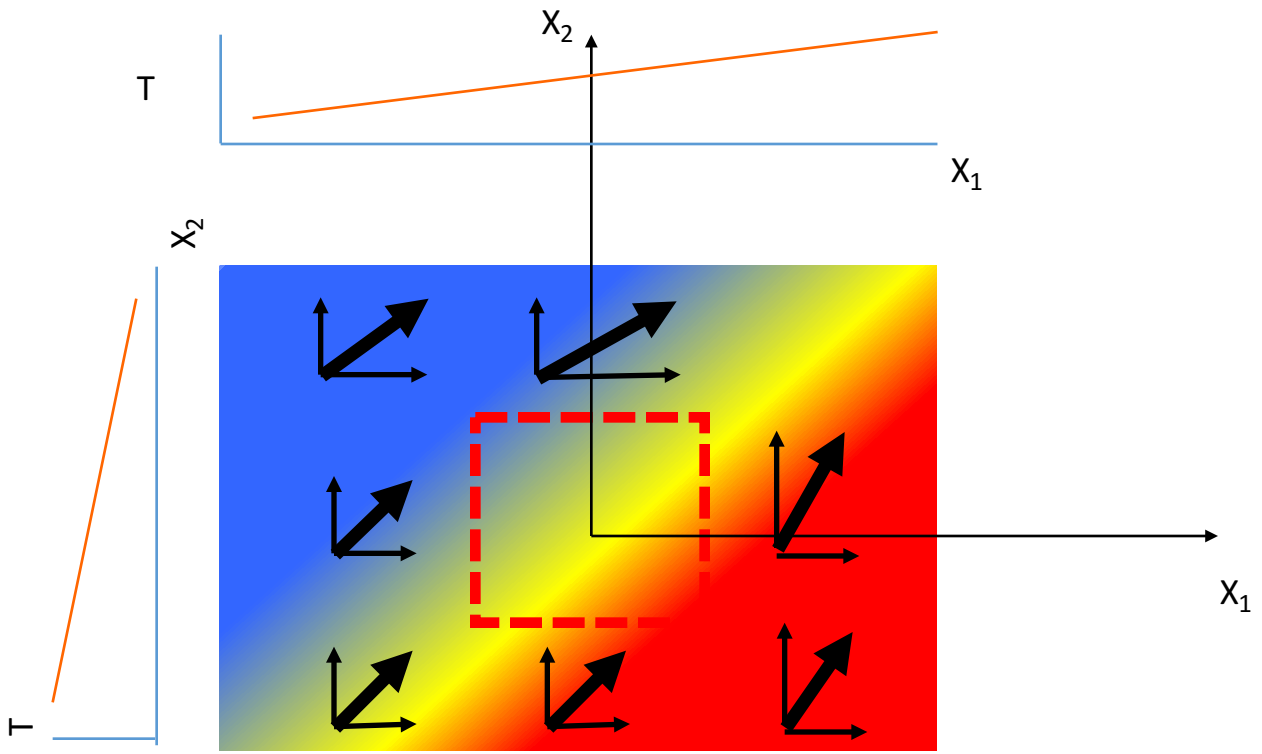
BONUS
ROUND 2

ANSWER



$\partial T/\partial t$ $\quad$ $\mathbf{u_1\,\partial T/\partial x_1}$ $\quad$ $\mathbf{u_2\,\partial T/\partial x_2}$ $\quad$ $\mathbf{\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)}$ $\quad$ $\mathbf{\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)}$ $\quad$ $(\partial u_1/\partial x_1)^2$ $\quad$ $(\partial u_2/\partial x_2)^2$ $\quad$ $\mathbf{[(\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2}$

BONUS ROUND 3



$\partial T/\partial t$

$u_1\ \partial T/\partial x_1$

$u_2\ \partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\ \partial T/\partial x_1)$

$\frac{\partial}{\partial x_2}(K\ \partial T/\partial x_2)$

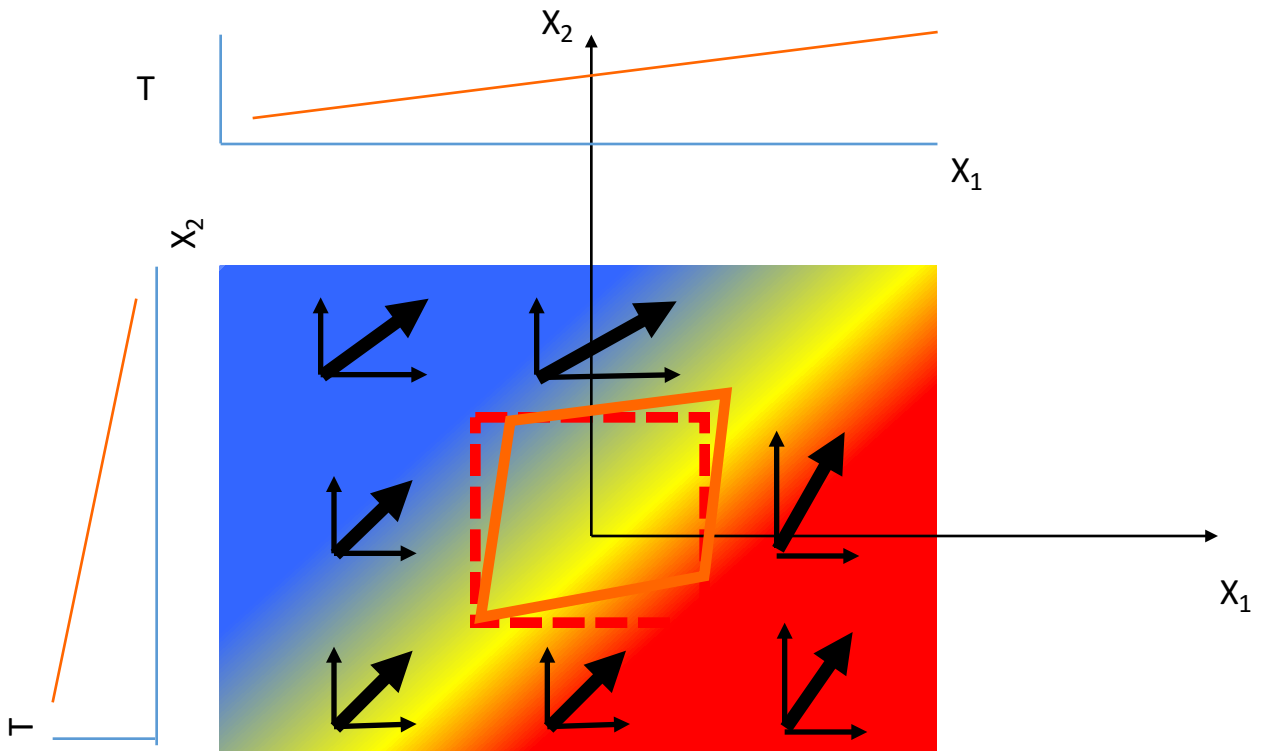
$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

BONUS
ROUND 3

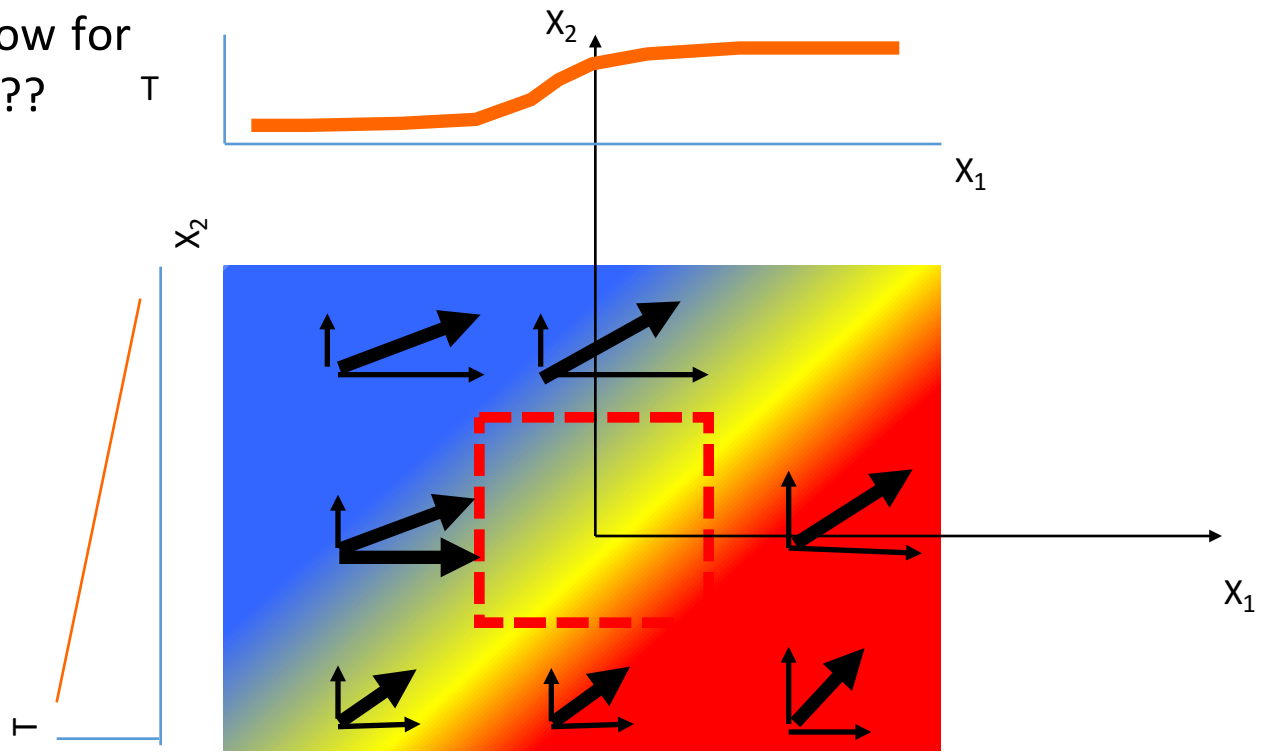
ANSWER



$\partial T/\partial t$ $\quad$ $u_1\,\partial T/\partial x_1$ $\quad$ $u_2\,\partial T/\partial x_2$ $\quad$ $\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$ $\quad$ $\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$ $\quad$ $(\partial u_1/\partial x_1)^2$ $\quad$ $(\partial u_2/\partial x_2)^2$ $\quad$ $[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

T(x_1) change….
What now for
dT/∂t????

BONUS
ROUND 4



∂T/∂t

u_1 ∂T/∂x_1

u_2 ∂T/∂x_2

$\frac{\partial}{\partial x_1}$ (K ∂T/∂x_1)

$\frac{\partial}{\partial x_2}$ (K ∂T/∂x_2)

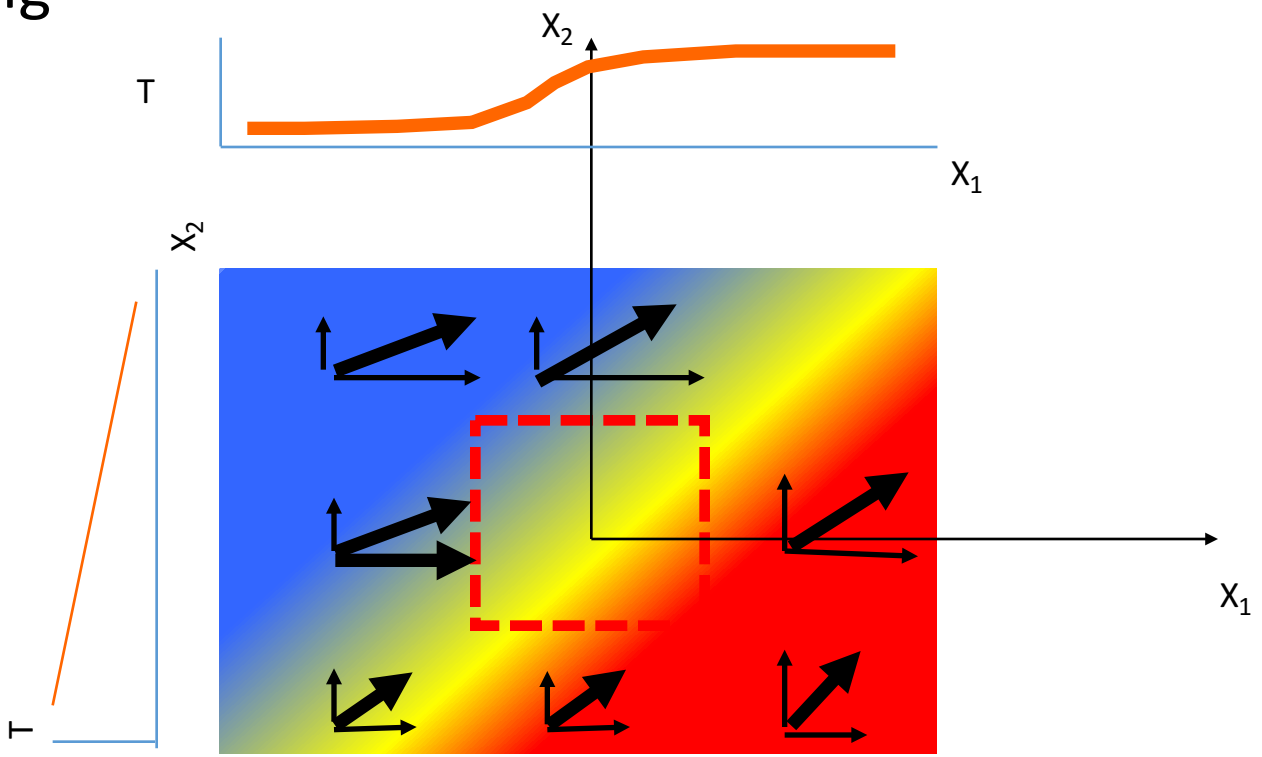
$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

Cooling

BONUS
ROUND 4



$\partial T/\partial t$!!!

$u_1\, \partial T/\partial x_1$

$u_2\, \partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$

$\frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

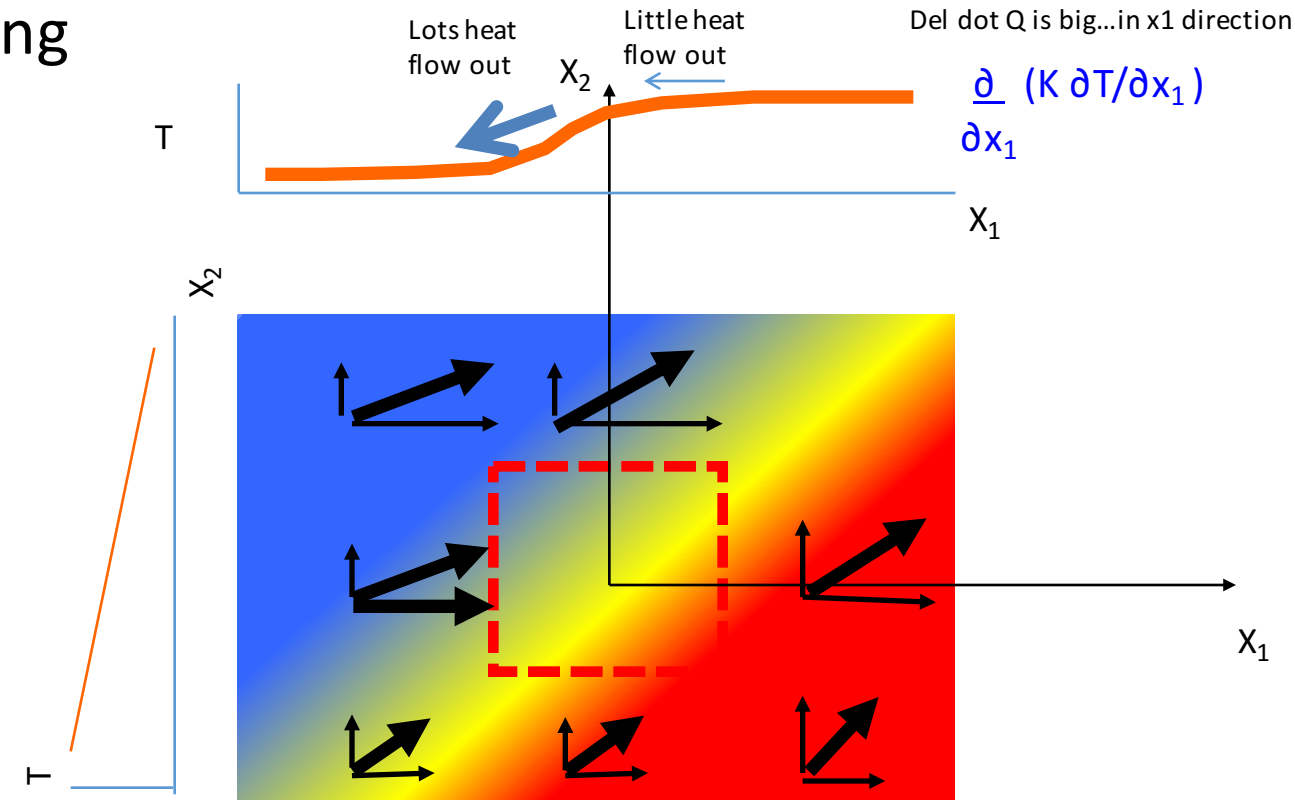
$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

Cooling

BONUS
ROUND 4



$\partial T/\partial t$!!!

$u_1\,\partial T/\partial x_1$

$u_2\,\partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$

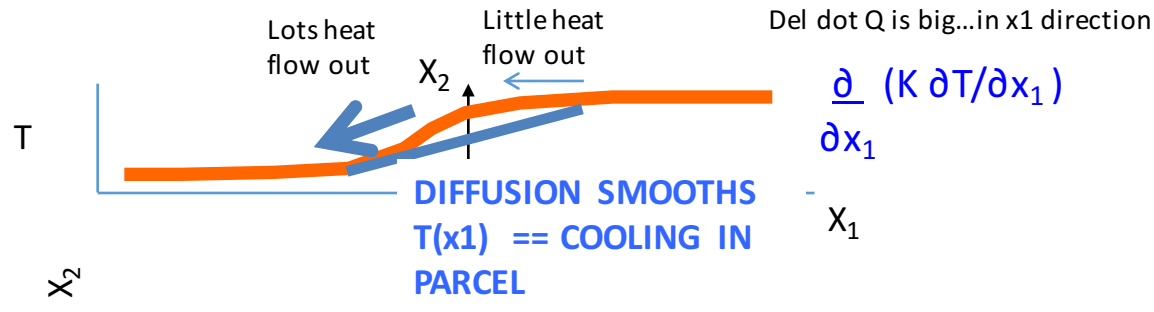
$\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$

$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

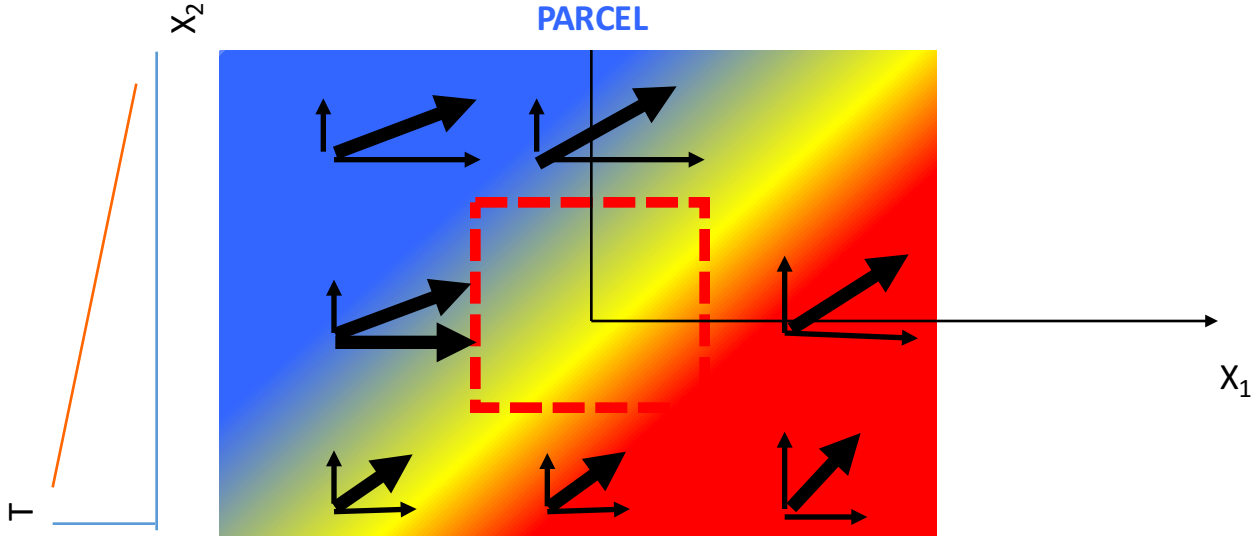
Cooling



Del dot Q is big…in x1 direction

$\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$

BONUS ROUND 4



$\partial T/\partial t$!!!

$u_1\,\partial T/\partial x_1$

$u_2\,\partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\,\partial T/\partial x_1)$

$\frac{\partial}{\partial x_2}(K\,\partial T/\partial x_2)$

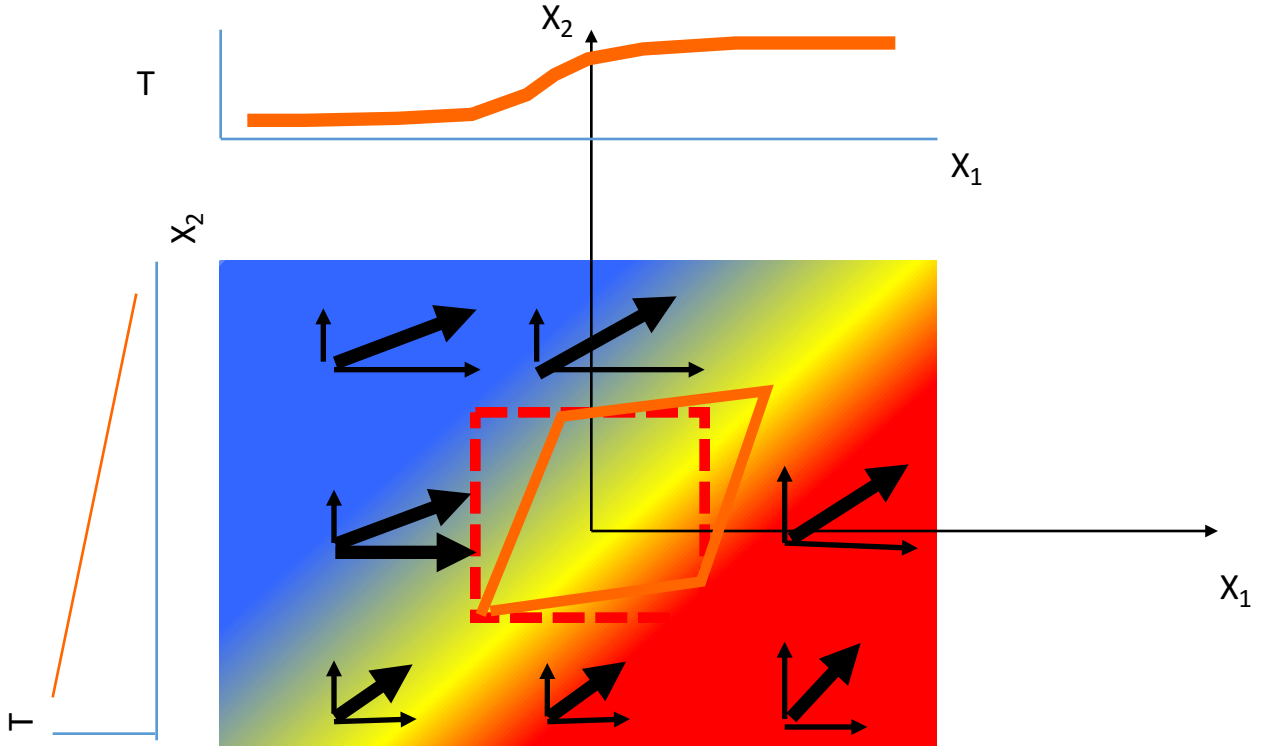
$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

LESS COOLING DUE TO WORK

BONUS
ROUND 4



$\partial T/\partial t$!!!

$u_1\, \partial T/\partial x_1$

$u_2\, \partial T/\partial x_2$

$\frac{\partial}{\partial x_1}(K\, \partial T/\partial x_1)$

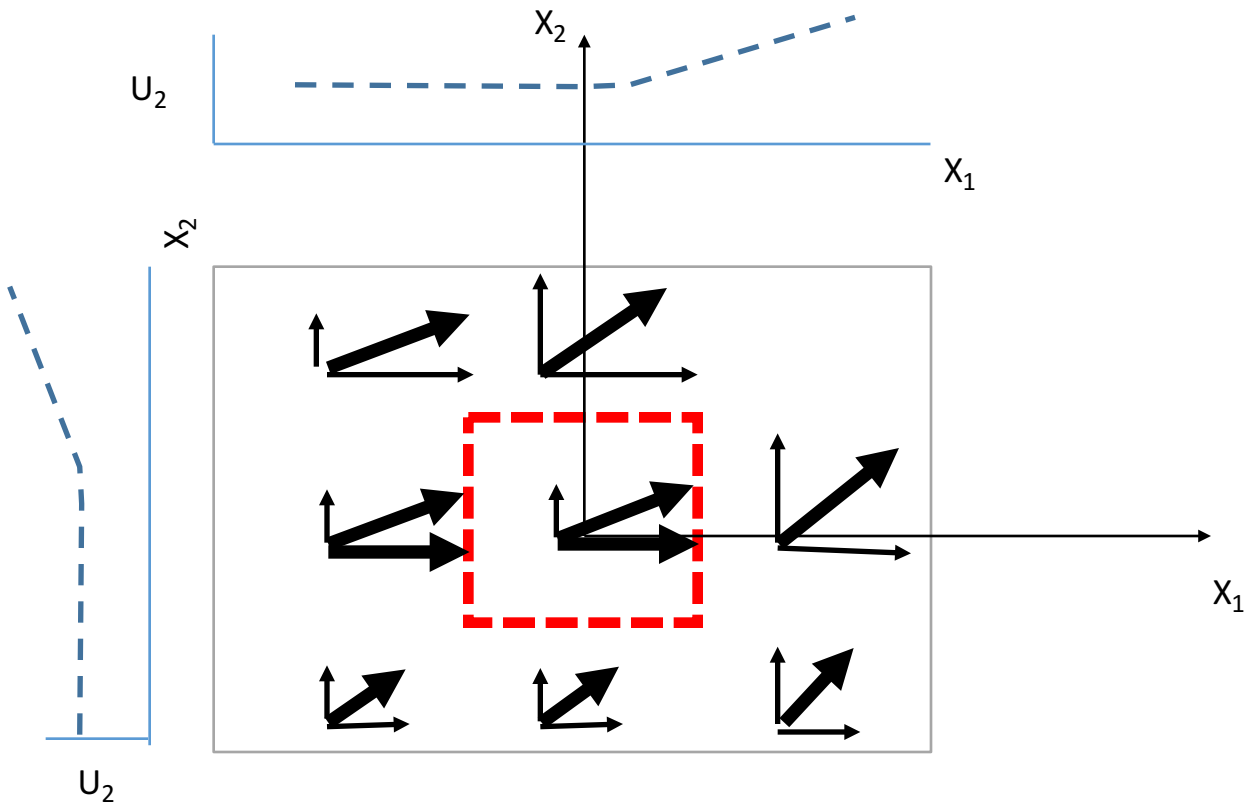
$\frac{\partial}{\partial x_2}(K\, \partial T/\partial x_2)$

$(\partial u_1/\partial x_1)^2$

$(\partial u_2/\partial x_2)^2$

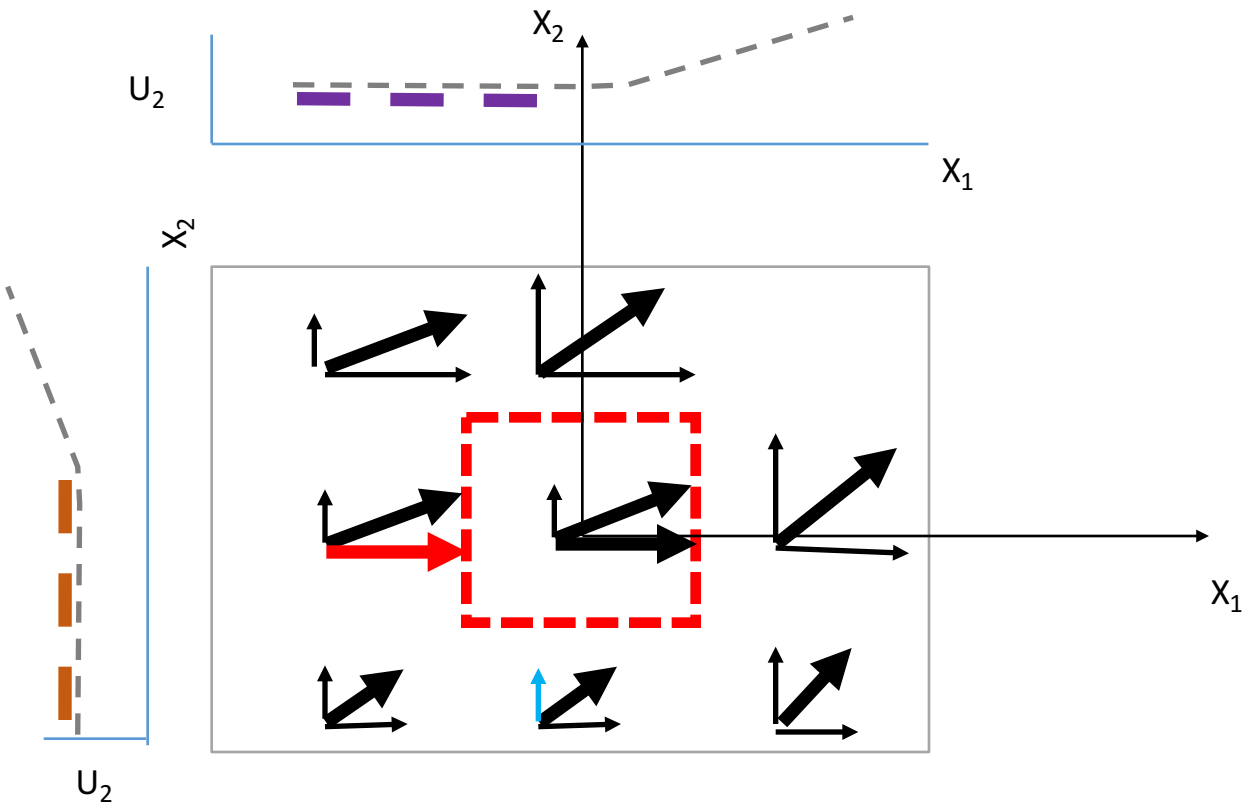
$[\partial u_1/\partial x_2)^2 + (\partial u_2/\partial x_1)^2$

Momentum Group 1



$\partial u_2/\partial t$ $u_1\, \partial u_2/\partial x_1$ $u_2\, \partial u_2/\partial x_2$ $\frac{\partial}{\partial x_1}(\mu\, (\partial u_2/\partial x_1))$ $\frac{\partial}{\partial x_2}(\mu\, (\partial u_2/\partial x_2))$

Momentum
Group 1
ANSWER



$\partial u_2/\partial t$

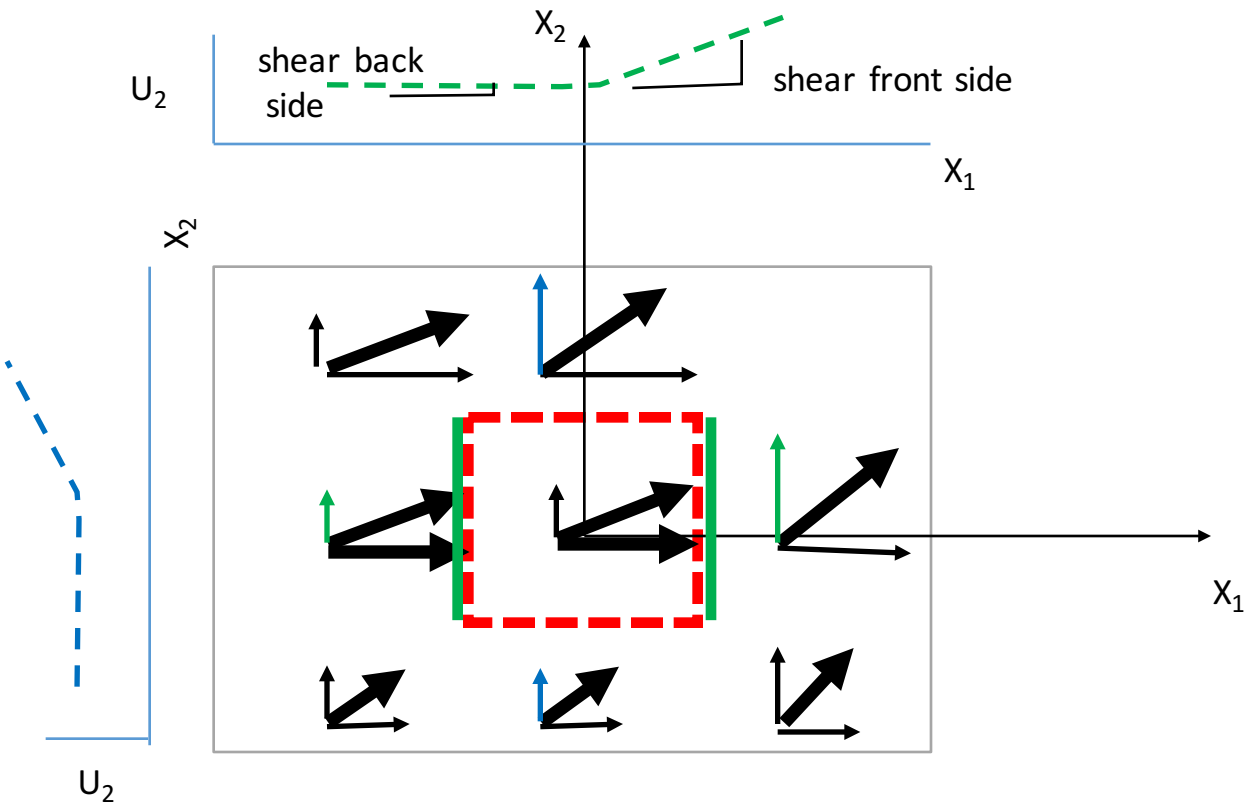
$\mathbf{u_1}\ \partial u_2/\partial x_1 = 0$

$\mathbf{u_2}\ \partial u_2/\partial x_2 = 0$

$\frac{\partial}{\partial x_1}(\mu(\partial u_2/\partial x_1))$

$\frac{\partial}{\partial x_2}(\mu(\partial u_2/\partial x_2))$

Momentum Group 1 ANSWER



grad of shear stress front – same at back

$\partial u_2/\partial t$

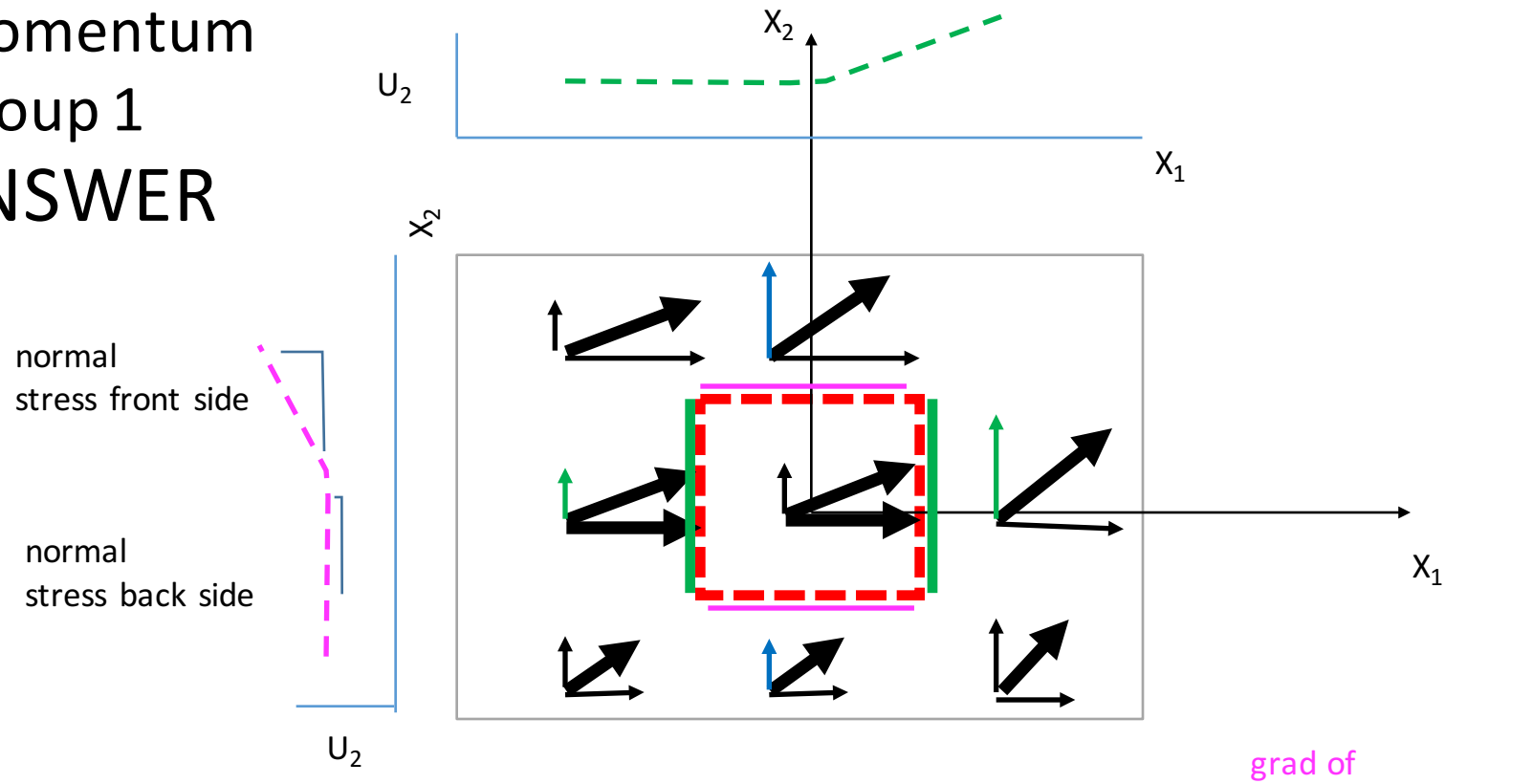
$u_1\, \partial u_2/\partial x_1$ =0

$u_2\, \partial u_2/\partial x_2$ =0

$\frac{\partial}{\partial x_1}(\mu(\partial u_2/\partial x_1))$ not 0

$\frac{\partial}{\partial x_2}(\mu(\partial u_2/\partial x_2))$ not 0

Momentum
Group 1
ANSWER



grad of shear stress front – same at back

grad of normal stress front – same at back

$\partial u_2/\partial t$

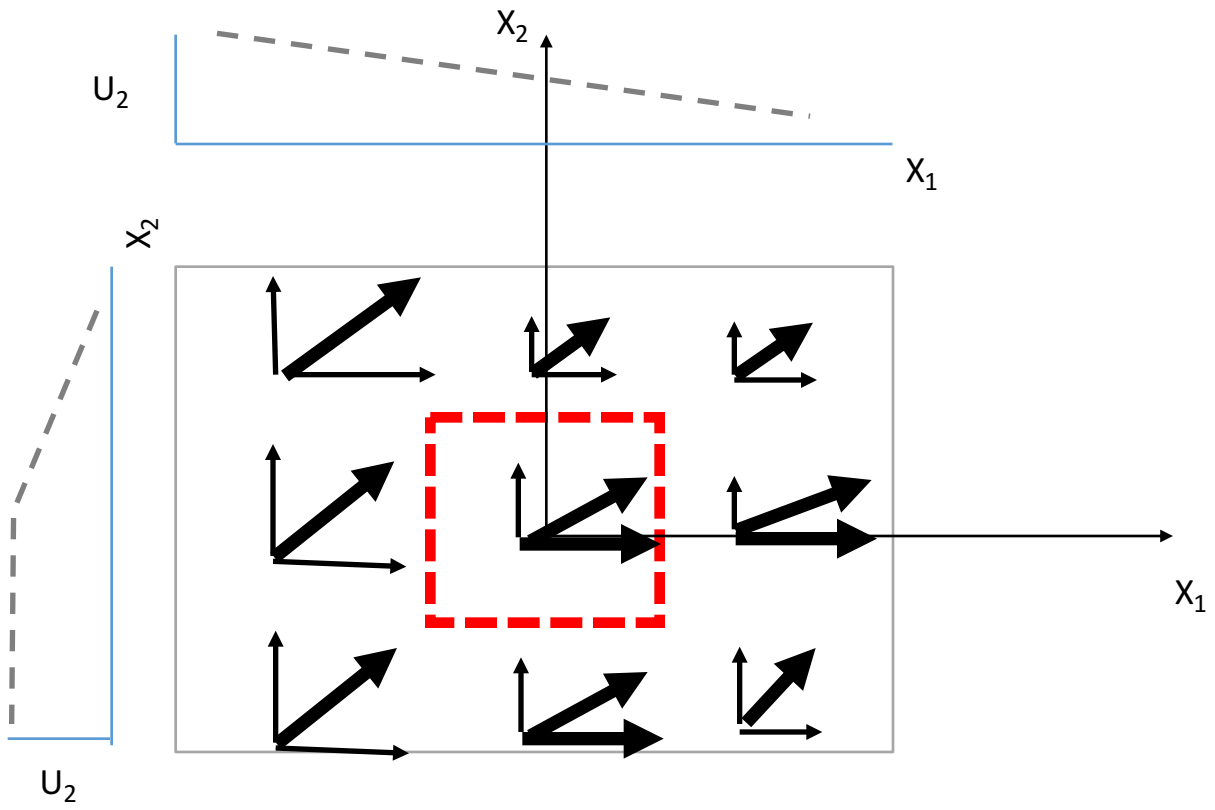
$u_1\, \partial u_2/\partial x_1$ =0

$u_2\, \partial u_2/\partial x_2$ =0

$\frac{\partial}{\partial x_1}(\mu(\partial u_2/\partial x_1))$ not 0

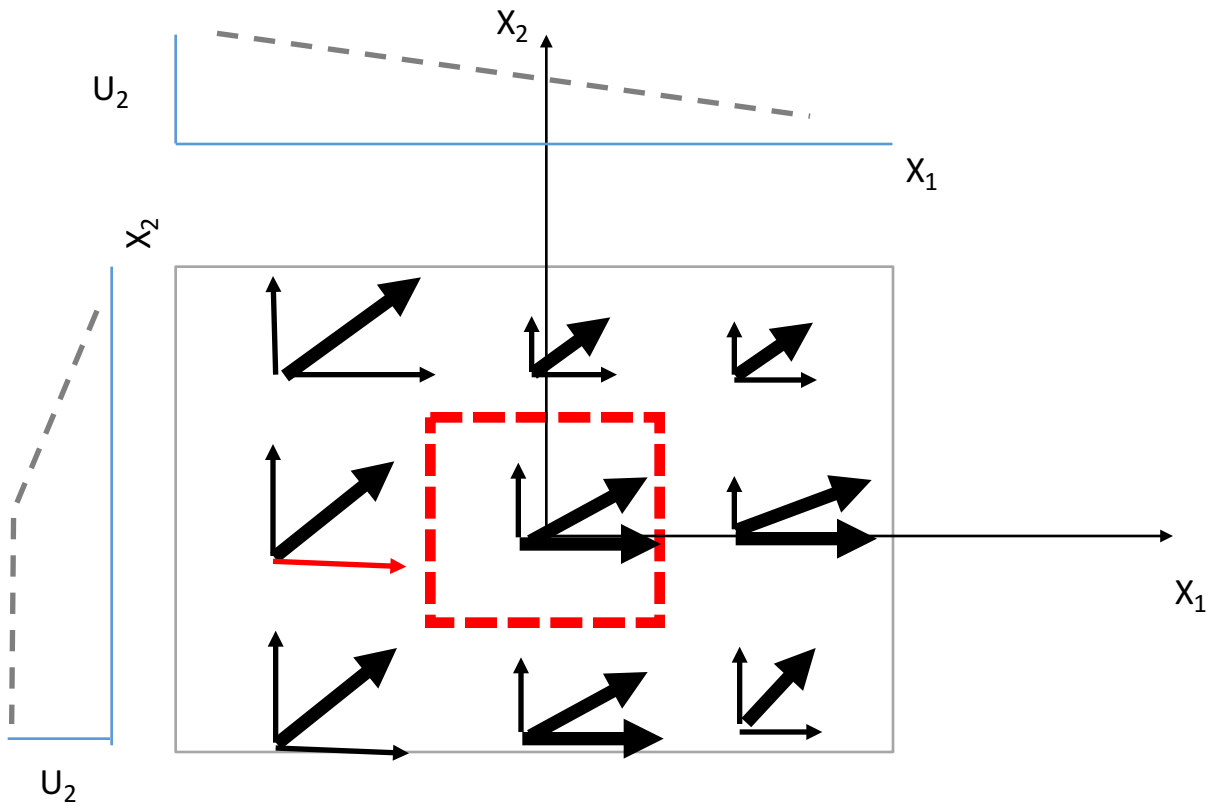
$\frac{\partial}{\partial x_2}(\mu(\partial u_2/\partial x_2))$ not 0

Momentum Group 2



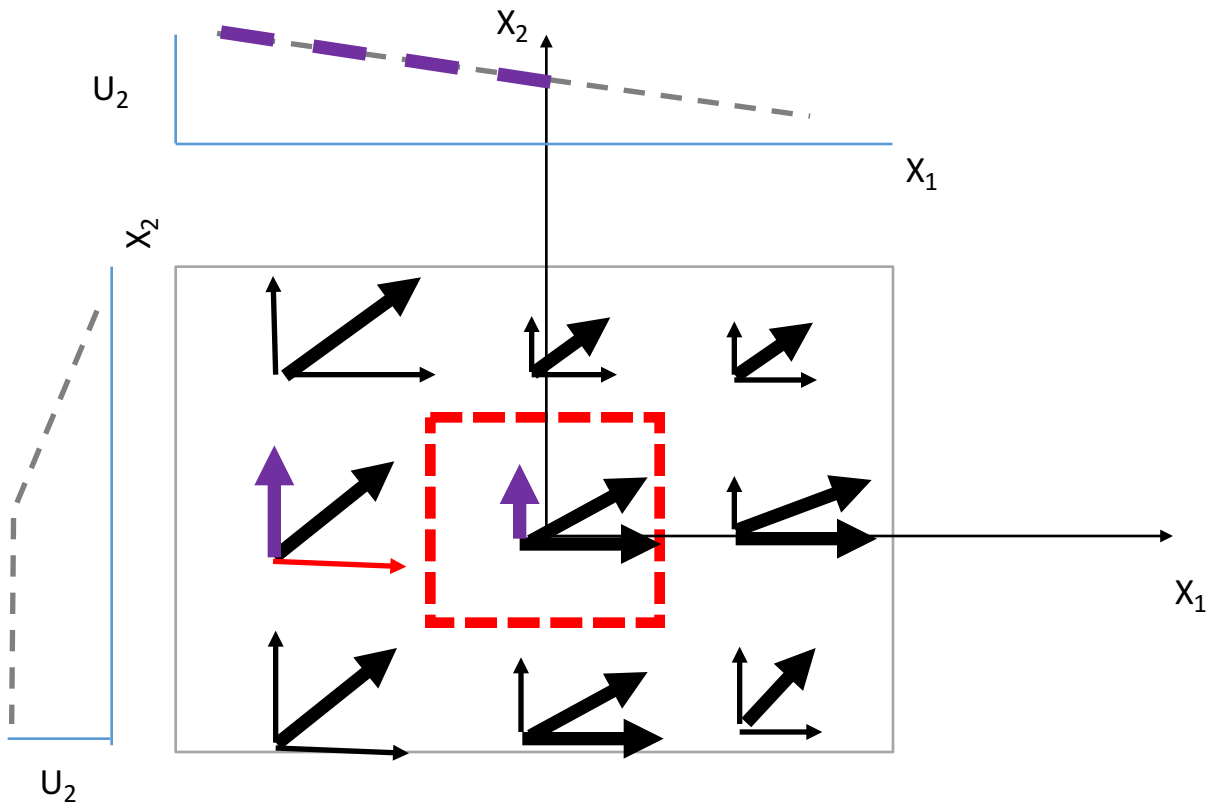
$\partial u_2/\partial t$ $u_1 \partial u_2/\partial x_1$ $u_2 \partial u_2/\partial x_2$ $\frac{\partial}{\partial x_1}(\mu(\partial u_2/\partial x_1))$ $\frac{\partial}{\partial x_2}(\mu(\partial u_2/\partial x_2))$

Momentum Group 2 ANSWER



$\partial u_2/\partial t$ $\quad$ $u_1\, \partial u_2/\partial x_1$ $\quad$ $u_2\, \partial u_2/\partial x_2$ $\quad$ $\frac{\partial}{\partial x_1}(\mu\, (\partial u_2/\partial x_1))$ $\quad$ $\frac{\partial}{\partial x_2}(\mu\, (\partial u_2/\partial x_2))$

Momentum Group 2 ANSWER



$\partial u_2/\partial t$

$u_1 \partial u_2/\partial x_1$

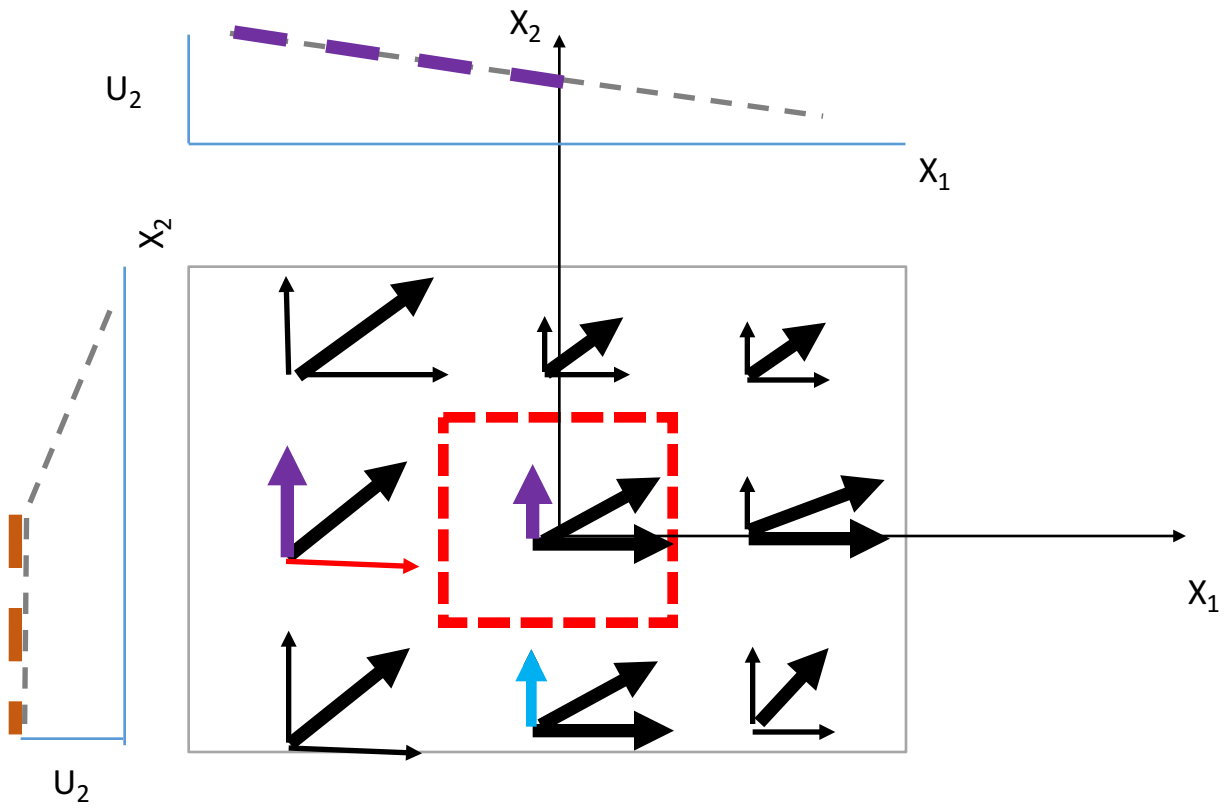
not 0

$u_2 \partial u_2/\partial x_2$

$\frac{\partial}{\partial x_1} (\mu (\partial u_2/\partial x_1))$

$\frac{\partial}{\partial x_2} (\mu (\partial u_2/\partial x_2))$

Momentum Group 2 ANSWER



$\partial u_2/\partial t$

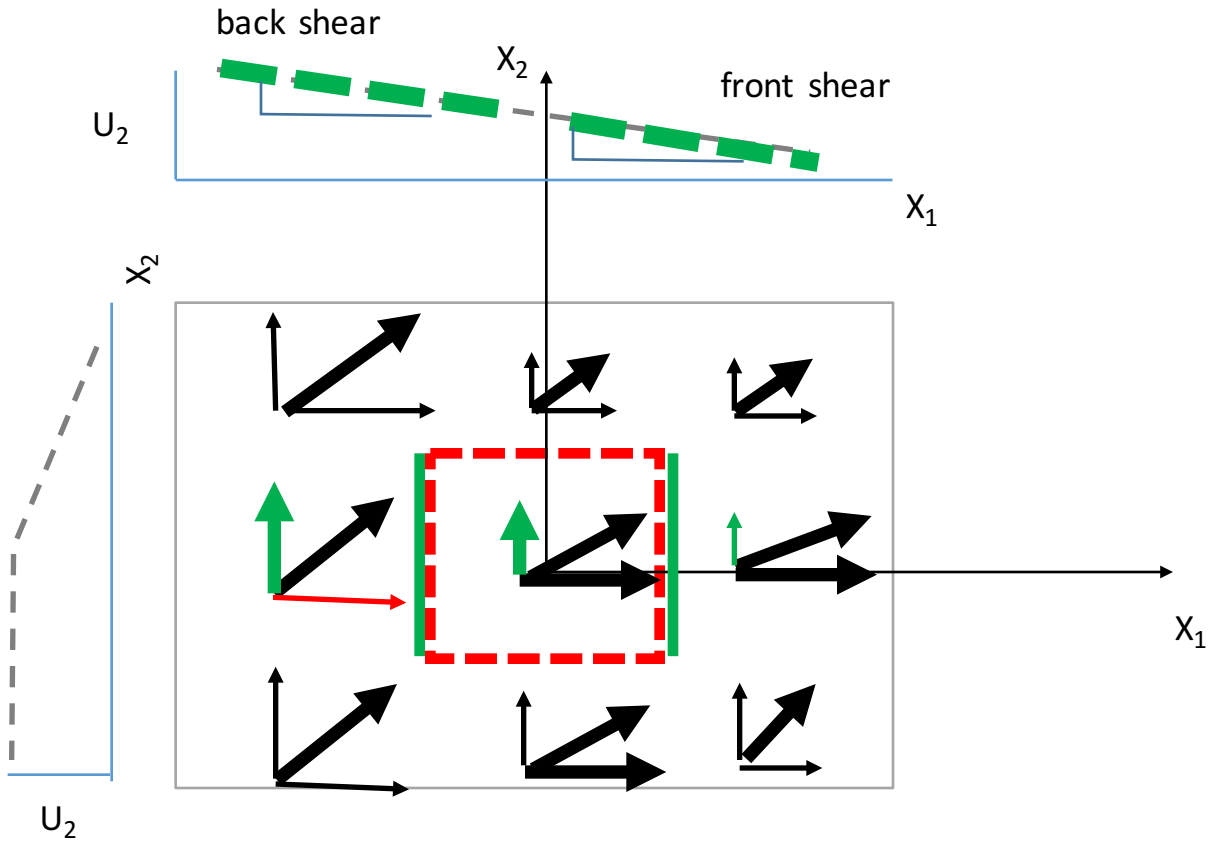
$u_1\, \partial u_2/\partial x_1$ not 0

$u_2\, \partial u_2/\partial x_2$ = 0

$\frac{\partial}{\partial x_1} (\mu (\partial u_2/\partial x_1))$

$\frac{\partial}{\partial x_2} (\mu (\partial u_2/\partial x_2))$

Momentum Group 2 ANSWER



grad of shear stress front – same at back

$\partial u_2/\partial t$

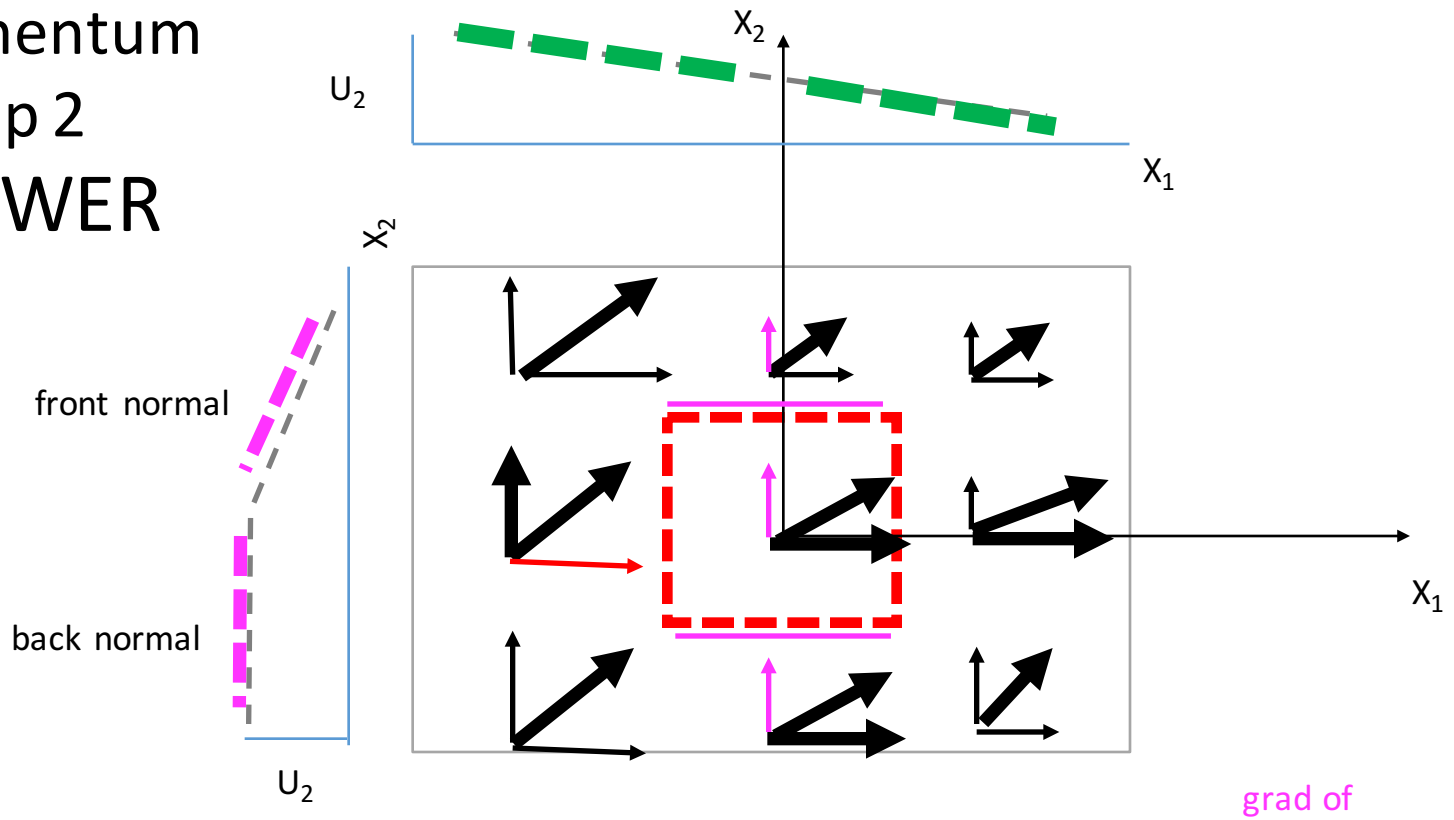
$u_1\, \partial u_2/\partial x_1$ not 0

$u_2\, \partial u_2/\partial x_2$ = 0

$\frac{\partial}{\partial x_1}(\mu\, (\partial u_2/\partial x_1))$ = 0

$\frac{\partial}{\partial x_2}(\mu\, (\partial u_2/\partial x_2))$

Momentum Group 2 ANSWER



grad of normal stress front – same at back

$\partial u_2/\partial t$

$u_1\, \partial u_2/\partial x_1$ increasing

$u_2\, \partial u_2/\partial x_2$ = 0

$\frac{\partial}{\partial x_1}(\mu(\partial u_2/\partial x_1))$ = 0

$\frac{\partial}{\partial x_2}(\mu(\partial u_2/\partial x_2))$ NOT 0, decreasing